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A Two-sector Marx-Kaleckian Model

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Abstract. In this paper we inherits Marx's reproduction schema and the idea of effective demand, and constructs a two-sector Marx-Kaleckian model with intermediate input by using method of Fujita(2019). This study investigates how the changes of mark-up rates affect the sectoral and macroeconomic capacity utilization and capital accumulation. It is found that in this model, Marx and Neo-Kaleckian basic judgments are still established. Meanwhile, the impact of one sector's change in the mark-up rate on performance differs by sector. We also calculate the equilibrium ratio of capital stocks.

1. **Introduction**

How changes in functional income distribution affect effective demand, economic fluctuations and economic growth has always been a hot topic in the Neo-Kaleckian model. In fact, this problem has been extensively studied in Neo-Kaleckian model whose setting is the single-sector and single-commondity. (Rowthorn, 1981; Dutt, 1984; Taylor 2004; Lavoie, 2014) Many theorical studies (Blecker, 2002; Lavoie and Stockhammer, 2013) distingushed two expansion regimes, *i.e.* profit driven and wage driven. While many empirical studies have examined whether the economy exhibits a wage-led or profit-led regime. For example, see Bowles and Boyer (1995), Stockhammer and Onaran (2004) and Storm and Naastepad (2012).

However, in real economies, market structure and labuor bargaining-power vary by industry (Calmfors *et al.*, 1988; Soskice, 1990; Lodovici, 2000), and accordingly how sectoral income distribution and degree of monopoly affects sectoral as well as macroeconomic performance should be considered. Dutt(1987) and Lavoie and Ramírez-Gastón (1997) use a two-sector model to investigate how a change in the sectoral mark-up rate influences economic performance.

The production of goods and services in any economy relies on a complex web of transactions between a wide range of suppliers and customers.[1] The Sraffian school criticizes that

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[1]As far back as the 1940s, in Leontief's study of the structure of the American economy, Leontief(1941) observed that almost everyone is "[...] aware of the existence of some kind of interconnection between even the remotest parts of a national economy" and that "the presence of these invisible but nevertheless very real ties can be observed whenever expanded automobile sales in New York City increase the demand for groceries in Detroit, [...] when the sudden shutdown of the Pennsylvania coal mines paralyzes the textile mills in New England," and in fact "with relentless regularity in alternative ups and downs of business cycles."

most Kaleckian models abstract from the input-output structure and the interdependence of industries (Steedman, 1992). Skott (2012) argued that the Kaleckian models cannot match the movements in the rate of capacity utilization for some individual industries. Franke (2000) introduce the intermediate input in the two-sector Kaleckian model, and consider the optimal use of capital and the financial sector in the price adjustment economy to conduct the stability analysis. In Franke(2000)'s model, the investment goods sector produces intermediate input goods as well as investment goods. Fujita(2019) followed the mothod of Franke(2000) and consider the effect of the mark-up rate. However, former work did not have microfoundation of the rate of capacity utilization, latter work cannot explain why the consumption good can become intermediate input goods .

This paper inherits the thought of Marx's reproduction schema and the effective demand of Neo-Kaleckian model, and constructs a two-sector Marx-Kaleckian model including intermediate-input by using Fujita's method(2019). It tries to answer the following three questions: First, whether the Neo-Kaleckian model and Marx's basic judgment of economic reality can be hold after expanding from a single sector to two sectors. Second, can the Marx-Kaleckian model of the two sectors reveal how sectors interact with each other? Third, using the two-sector Marx-Kaleckian model, can we find the mechanism of how the change of mark-up rate affects the macroeconomy?

The remainder of the paper is organized as follows. Section 2 presents a two-sector model. Section 3 assumes that capital stocks do not accumulate and investigates the short-run effect of a change in the mark-up rate on the rate of capacity utilization. Section 4 assumes that capital stocks accumulate and investigates the long-run effect of a change in the mark-up rate on capital accumulation and calculates the equilibrium of the sectoral ratio of capital stocks. Section 5 concludes.

2. **Model**

The assumptions of the model are as follows:

(1) In this paper, we consider a closed economy without goverment. The economy is composed of an investment goods sector (sector 1) and a consumption goods sector (sector 2). The product of sector 1 is used as intermediate inputs and fixed capital. The product of sector 2 can only be consumed.

(2) The production function of each sector is a Leontief production function with fixed coefficients. Both sectors produce good by using fixed capital stocks, intermediate input goods, and labour.

(3) The fixed capital cannot be moved among sectors in the short run. On the contrary, labour is movable among sectors. In the long run, The fixed capital and labour can move freely.

(4) According to opinion of Marx and Kalecki, there are no labour supply constraints for the existenc of the reserve army of labour.

(5) In both sectors, the firms adopt mark-up pricing,[2] and have excess capacity.
(6) The net products are distributed to workers as wages and to capitalists as profits. Workers spend all their wage income on consumption goods, whereas capitalists save a constant proportion of their profit income to invest.
(7) For the sake of simplicity, we assume that there is no technical progress and there is no depreciation of fixed capital stocks in this model.

According to the model setting, in each sector the firm prices its good by mark-up pricing .For the sake of simplicity, we assume the firm prices its good following the pattern:

$$p_1 = (1 + \theta_1)(a_{11} p_1 + W b_1), \tag{2.1}$$

$$p_2 = (1 + \theta_2)(a_{12} p_1 + W b_2), \tag{2.2}$$

where p_i $(i = 1, 2)$ denotes the price of sector i, $\theta_i (i = 1, 2)$ denotes the mark-up rate, W denotes the nominal wage of society which is equal between sector 1 and sector 2, a_{ij} denotes the coefficient which is the quantity of good i used for production of one unit of good j, b_i the labour input coefficient in sector i. θ_i, a_{ij}, b_i are positive number. According to the setting of Marxian reproduction schema, the consumer good cannot be used as intermediate inputs, it is only used to consume for maintaining the reproduction of labor, *i.e.* $a_{21} = a_{22} = 0$; the mark-up rate (θ_i) reflect the monopoly power in the market. The higher the monopoly power or stronger the bargaining power of the firm, the higher the mark-up rate is (Kalecki, 1971; Sen and Dutt,1995).

Next, we talk about the sector 1. For that the investment good and intermediate inputs in this economy are all produced by sector 1, the output of sector 1 must satisfy following equation:

$$p_1 X_1 = p_1 a_{11} X_1 + p_1 a_{12} X_2 + p_1 (I_1 + I_2), \tag{2.3}$$

where X_1 denotes the output of sector 1, $I_i$denotes the investment of sector i, $a_{ij} X_j$ denotes the intermediate inputs used by sector j. LHS of equation (2.3) means that the investment good and intermediate inputs in this economy are all produced by sector 1. The RHS of equation (2.3) means that the investment good are used as investment and intermediate inputs.

The supply of consumption goods meets demand as follows:

$$p_2 X_2 = p_2 C, \tag{2.4}$$

where C denotes the consumption of society. According to the model setting, the consumption of workers all comes from the wage income, capitalists save a constant proportion of their profit income. We decompose the the consumption of society as follows:

$$\begin{aligned} p_2 C =& W (b_1 X_1 + b_2 X_2) \\ &+ (1 - s) [\theta_1 (a_{11} p_1 + W b_1) X_1 + \theta_2 (a_{12} p_1 + W b_2) X_2]. \end{aligned} \tag{2.5}$$

[2]See Lavoie (1992), Lee (1998) and Coutts and Norman (2013) on post-Keynesian price theory.

The first term in RHS of equation (2.5) is the consumption of the worker, whilst the second term the consumption of the capitalist.

The relationship among the economy can be represented by the input-output table (1):

TABLE 1. Input-Output Table

		Intermediate demand		final demand		output
		sector 1	sector 2	consumption	investment	
intermediate	sector 1	$a_{11}p_1X_1$	$a_{12}p_1X_2$	0	$p_1(I_1+I_2)$	p_1X_1
inputs	sector 2	0	0	p_2C	0	p_2X_2
value added	Wage	Wb_1X_1	Wb_2X_2			
	Profit	$\theta_1(a_{11}p_1+Wb_1)X_1$	$\theta_2(a_{12}p_1+Wb_2)X_2$			
input		p_1X_1	p_2X_2			

We take commodity 2 (consumer goods) as the numeraire, normalise it to 1, and calculate the relative price ($p \equiv \frac{p_1}{p_2}$) and real wages ($w \equiv \frac{W}{p_2}$) in the model by (2.1) and (2.2):

$$p \equiv \frac{p_1}{p_2} = \left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-1} \frac{1}{b_2(1+\theta_2)}, \tag{2.6}$$

$$w \equiv \frac{W}{p_2} = \left[1 - \frac{a_{12}}{b_2}\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-1}\right] \frac{1}{b_2(1+\theta_2)}. \tag{2.7}$$

To make the equation has economic meaning, *i.e.* $p > 0, w > 0$, we need parameters $(a_{11}, a_{12}, b_1, b_2, \theta_1)$ to satisfy the following conditions:

$$\begin{gathered} -\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1} > 0 \\ \frac{a_{12}}{b_2}\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-1} < 1. \end{gathered} \tag{2.8}$$

We take the partial derivatives of the relative price and real wages with respect to mark-up rate (θ_1, θ_2),

$$\begin{aligned}
\frac{\partial p}{\partial \theta_1} &= \left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-2} \left[\frac{1}{b_2(1+\theta_2)}\right] \frac{1}{b_1(1+\theta_1)^2} > 0 \\
\frac{\partial p}{\partial \theta_2} &= \left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-1} \left[\frac{-1}{b_2(1+\theta_2)^2}\right] < 0 \\
\frac{\partial w}{\partial \theta_1} &= -\frac{1}{b_2(1+\theta_2)}\frac{a_{12}}{b_2}\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-2} \frac{1}{b_1}\left(\frac{1}{1+\theta_1}\right)^2 < 0 \\
\frac{\partial w}{\partial \theta_2} &= \left[1 - \frac{a_{12}}{b_2}\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)^{-1}\right] \frac{-1}{b_2(1+\theta_2)^{-2}} < 0.
\end{aligned}$$

From the results above, we find that a rise in the mark-up rate in sector 1 raises the relative price and reduces the real wage. A rise in the mark-up rate in sector 2 reduces both the relative price and the real wage. A rise in θ_1 increases the consumption goods price when we consider the intermediate inputs then reduces the real wage.

$$m_1 = 1 - \frac{wb_1}{(1-a_{11})\,p} = \frac{\theta_1}{(1-a_{11})(1+\theta_1)}, \tag{2.9}$$

$$m_2 = 1 - \frac{wb_2}{1-a_{12}p} = \frac{\theta_2}{(1-a_{12}p)(1+\theta_2)}. \tag{2.10}$$

We know that if conditions (2.8) hold, the profit share (m_1, m_2) in both sectors are positive and smaller than unity. In order to find how the change of mark-up rate affects profit share, we take the partial derivatives of profit-shares (m_1, m_2) with respect to the mark-up rates (θ_1, θ_2):

$$\frac{\partial m_1}{\partial \theta_1} = \frac{1}{(1-a_{11})(1+\theta_1)^2} > 0, \tag{2.11}$$

$$\frac{\partial m_2}{\partial \theta_1} = \frac{1}{(1+\theta_1)^2 b_1} \frac{\frac{a_{12}}{b_2}\left(1 - \frac{1}{(1+\theta_2)}\right)}{\left[(1+\theta_2)\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right) - \frac{a_{12}}{b_2}\right]^2} > 0, \tag{2.12}$$

$$\frac{\partial m_1}{\partial \theta_2} = 0, \tag{2.13}$$

$$\frac{\partial m_2}{\partial \theta_2} = \frac{\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)\left(-\frac{a_{11}}{b_1} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right)}{\left[(1+\theta_2)\left(-\frac{a_{11}}{b_1} + \frac{a_{12}}{b_2} + \frac{1}{b_1}\frac{1}{1+\theta_1}\right) - \frac{a_{12}}{b_2}\right]^2} > 0. \tag{2.14}$$

From results above, we find that profit share of each sectors has a postive relation with its own mark-up rate. Because the higher mark-up rate (θ_i) in its own sector reflects the higher monopoly degree of firms in the sector and its stonger bargaining power in the distribution of wages and profits, a higher mark-up rate (θ_i) will bring a higher profit share within the sector. So the positive relationship between mark-up rate and profit share still holds.

Unlike the traditional Neo-Kaleckian model, the mark-up rate (θ_1) of sector 1 in this model has spillover effect. By observing the above results, we can see that the increases of mark-up rate of sector 1 (θ_1) will induce the increase of profit share in two sectors. The traditional Neo-Kaleckian model assumes that the intermediate input coefficient $a_{ij} = 0$, so the increase of mark-up rate of a sector does not affect the profit share of the other sector. In our model, the increase of mark-up rate of sector 2 still has no effect on the profit share of the other sector for the increase of the mark-up rate of sector 1 will increase the profit share of sector 1 and sector 2. The increase of the mark-up rate of sector 1 reduces the real wage rate and raises the relative price. The effect of reducing the real wage will increase the profit share of sector 2. The increase of the relative price of sector 1 will decrease the profit share of sector 2. The former effect exceeds the latter effect and leads to the increase of the profit share of sector 2. However, the increase of the mark-up rate of sector 2 does not affect the profit share of sector 1. This is because the increase of the mark-up rate of sector 2 reduces the real wage rate and the relative price at the same time, and the influence of the former is offset by the influence of the latter, which leads to the unchanged profit share of sector 1.

Short-term equilibrium is reflected in the equilibrium of flow which means investment equals savings in the economy. In order to solve the short-term equilibrium growth rate, we need to discuss the relationship between capital accumulation rate and savings growth rate in the model.

The investment function in the New Cambridge School reflects the relationship between investment and the profit rate. The New Cambridge School believes that the future is unpredictable, so capitalists' judgment on whether investment can be profitable mainly comes from past profitability. Therefore, the investment function is mainly determined by "animal spirit" and the profit rate (Robinson, 1962). Rowthorn (1981) and Dutt (1984) assume that the investment function depends not only on the profit rate, but also on capacity utilization. Bhaduri and Marglin (1990), Lavoie and Ramrlez-Gaston (1997) pointed out that it was not appropriate to put both productivity utilization rate and the profit rate into the investment function. This assumption actually imposes unnecessary restriction: investment must be more sensitive to changes in capacity utilization than investment to changes in the profit share. Thus, profit-driven mechanism is excluded from the model. They put profit share and capacity utilization into the investment function as independent variables. In order to solve the problem conveniently, Blecker (2002) further expressed the investment function by a simple linear relation. Thus, we adopt the Bhaduri-Marglin-type investment function as follows:

$$g_1 \equiv \frac{I_1}{K_1} = \alpha_1 + \beta_1 m_1 + \gamma_1 u_1, \tag{2.15}$$

$$g_2 \equiv \frac{I_2}{K_2} = \alpha_2 + \beta_2 m_2 + \gamma_2 u_2, \tag{2.16}$$

where, g_i represents the capital accumulation rate of sector i, K_i represents the capital stock of sector i, $k \equiv K_1/K_2$ denotes the sectoral ratio of capital stocks, $u_i \equiv X_i/K_i$ represents the capacity utilization rate of sector i; α_i represents the capitalists's animal spirit in sector i; β_i represents coefficient of investment to the profit share, the γ_i are coefficient of investment to capacity utilization. α_i, β_i, and γ_i are positive parameters.

Dividing the left hand side and right hand side of equation (2.3) by $p_1 K_1$, we can obtain

$$u_1 = a_{11} u_1 + a_{12} \left(\frac{X_2}{K_2}\right) \left(\frac{K_2}{K_1}\right) + g_1 + g_2 \left(\frac{K_2}{K_1}\right). \tag{2.17}$$

Substituting equation (2.15) and (2.16) and rearranging, we obtain that:

$$u_1 = \frac{\alpha_1 + \beta_1 m_1 + (\alpha_2 + \beta_2 m_2) k^{-1} + (a_{12} + \gamma_2) k^{-1} u_2}{1 - a_{11} - \gamma_1}. \tag{2.18}$$

By substituting equation (2.5) into (2.4), we obtain

$$p_2 X_2 = W (b_1 X_1 + b_2 X_2) + (1 - s) [\theta_1 (a_{11} p_1 + W b_1) X_1 + \theta_2 (a_{12} p_1 + W b_2) X_2 \tag{2.19}$$

Then, dividing the left hand side and right hand side of equation (2.19) by $p_2 K_2$ and substituting the equations of profit share of both sectors, we obtain that

$$u_2 = \frac{(1-m_1)(1-a_{11})\,pk + (1-s)(1-a_{11})\,m_1 pk}{1-(1-m_2)(1-a_{12}p)-(1-s)(1-a_{12})\,m_2} u_1, \tag{2.20}$$

we define a coefficient A:

$$A \equiv \frac{(1-m_1)(1-a_{11})\,pk + (1-s)(1-a_{11})\,m_1 pk}{1-(1-m_2)(1-a_{12}p)-(1-s)(1-a_{12})\,m_2},$$

and make an assumption that:

$$A = \frac{(1-m_1)(1-a_{11})\,pk + (1-s)(1-a_{11})\,m_1 pk}{1-(-m_2)(1-a_{12}p)-(1-s)(1-a_{12})\,m_2} < \frac{(1-a_{11}-\gamma_1)\,k}{a_{12}+\gamma_2}.$$

Now, we can obtain the capacity utilization rate of each sector (u_1^*, u_2^*) in the equilibrium:

$$u_1^* = \frac{\alpha_1 + \beta_1 m_1 + (\alpha_2 + \beta_2 m_2)\,k^{-1}}{1-a_{11}-\gamma_1 - Ak^{-1}(a_{12}+\gamma_2)}, \tag{2.21}$$

$$u_2^* = Au_1^*. \tag{2.22}$$

By substituting (2.21)–(2.22) to (2.15) and (2.16), we can obtain the capital accumulation rate of sectors:

$$g_1 \equiv \frac{I_1}{K_1} = \alpha_1 + \beta_1 m_1 + \gamma_1 \frac{\alpha_1 + \beta_1 m_1 + (\alpha_2 + \beta_2 m_2)\,k^{-1}}{1-a_{11}-\gamma_1 - Ak^{-1}(a_{12}+\gamma_2)}, \tag{2.23}$$

$$g_2 \equiv \frac{I_2}{K_2} = \alpha_2 + \beta_2 m_2 + \gamma_2 A \frac{\alpha_1 + \beta_1 m_1 + \alpha_2 + \beta_2 m_2}{1-a_{11}-\gamma_1 - \mathrm{A}k^{-1}(a_{12}+\gamma_2)}. \tag{2.24}$$

Analyzing the equilibrium growth rate of each sector, we can find that many Kalecki's ideas still hold in this model.

First, paradox of thrift is valid in this model. We take the partial deivatives of growth rates with respec to the saving rate (s)

$$\frac{\partial g_1}{\partial s} < 0,$$
$$\frac{\partial g_2}{\partial s} < 0.$$

The partial derivatives of the growth rate of each sectors with respect to s is negative. So the savings rate will restrain economic growth, and the paradox of thrift still holds. In the capitalist society, workers do not have enough income to purchase goods, so that products in the economy cannot be fully consumed, while the high savings of capitalists crowds out consumption, resulting in insufficient effective demand, and the speed of economic growth has been constrained.

Secondly, the mechanism of economic fluctuation caused by capitalists' "animal spirit" is still valid in this model. Keynes (1936) believed that the investment behavior of the main investor in the capitalist society, the capitalist, was susceptible to psychological factors such as "animal spirit" and had great fluctuation. Because of the investment multiplier effect in the economy, the fluctuation of investment magnified into the fluctuation of the overall economy.

We take the partial derivatives of the growth rate (g_i) of each sector with respect to saving rate (α_i):

$$\frac{\partial g_1}{\partial \alpha_1} = 1 + \frac{\gamma_1}{1 - a_{11} - \gamma_1 - \mathrm{A}k^{-1}\left(a_{12} + \gamma_2\right)} > 1 > 0$$
$$\frac{\partial g_1}{\partial \alpha_2} = 1 + \frac{\gamma_1 k^{-1}}{1 - a_{11} - \gamma_1 - \mathrm{Ak}^{-1}\left(a_{12} + \gamma_2\right)} > 0$$
$$\frac{\partial g_2}{\partial \alpha_1} = \frac{\gamma_2 A}{1 - a_{11} - \gamma_1 - \mathrm{A}k^{-1}\left(a_{12} + \gamma_2\right)} > 0$$
$$\frac{\partial g_2}{\partial \alpha_2} = 1 + \frac{\gamma_2 A}{1 - a_{11} - \gamma_1 - \mathrm{A}k^{-1}\left(a_{12} + \gamma_2\right)} > 1 > 0.$$

It can be seen that the output of either sector 1 or sector 2 changes in the same direction as the "animal spirit". If the capitalists of this sector are optimistic about the future economy, and the rise of autonomous investment will have a multiplier effect. Due to the existence of intermediary inputs and the free flow of workers among sectors, the increase of autonomous investment in this sector will also have spillover effects on other sector.

3. Short-Run Analysis

3.1. θ_1 **increase.** According to the relationship among m_1, m_2, p and θ_1, we can see that: firstly, because the monopoly power of sector 1 is strengthened, the increase of the mark-up rate of sector 1 reduces the real wage rate w and raises the relative price p, which increases the profit share of the investment sector. secondly, the effect of reducing the real wage will increase the profit share of sector 2, and the increase of the relative price of sector 1 will decreases the profit share of the sector 2. The former effect exceeds the latter effect, so the profit share of sector 2 increases. When other parameters remain unchanged, we find that in the (u_2, u_1) graph, the intercept of the line representing the equation (2.18) increases and the slope of the line representing the equation (2.18) remains unchanged. The line representing (2.20) only changes the slope. In order to find the new equilibrium point of productivity utilization (u_1^{**}, u_2^{**}) we need to discuss the change of constant A^{-1}.

However, the relationship between constants A^{-1} and m_1, m_2, p cannot be determined, there will be many situations:

- A^{-1} becomes larger. The raise of mark-up rate of sector 1 will have a dual effect: on the one hand, increase of the profit share of the two sectors will reduce the consumption of workers, resulting in insufficient effective demand, poor commodity sales and idle factory capital, which we can call "consumer demand effect". On the other hand, increase of profit share will increase capitalists' investment expenditure, make economic expansion, which we can call it "investment demand effect". There will be many kinds of equilibrium possibilities. The impact of "investment demand effect" on the economy exceeds that of "consumption demand effect", and the economy will expand. The new equilibrium point of capacity utilization (u_1^{**}, u_2^{**}) is shown as $(u_1^{**} > u_1^*, u_2^{**} > u_2^*)$. The "consumption demand effect" is stronger than "investment demand effect", which

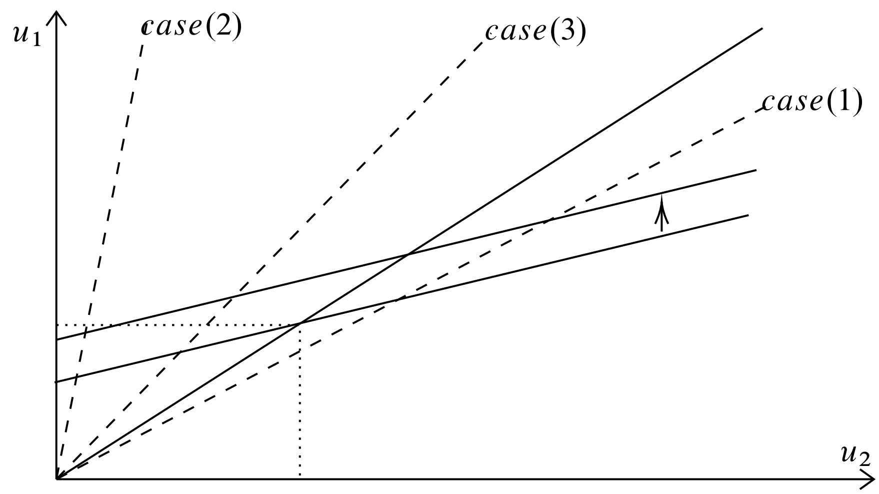

FIGURE 1. Three Expansion Regimes

will make the equation (2.20) The counter-clockwise rotation greater. The new equilibrium point of capacity utilization (u_1^{**}, u_2^{**}) is shown as $(u_1^{**} > u_1^*, u_2^{**} < u_2^*)$. If the former profit share m_1, m_2 is too large, the "consumption demand effect" will be much stronger than the "investment demand effect" and even the equilibrium point $(u_1^{**} < u_2^*)$.

- A^{-1} becomes smaller. In this situation, the impact of "investment demand effect" on the economy exceeds that of "consumption demand effect", and the economy will expand. The new equilibrium point of capacity utilization (u_1^{**}, u_2^{**}) is shown as $(u_1^{**} > u_1^*, u_2^{**} > u_2^*)$.

3.2. θ_2 **increase.** Next, we consider the impact of the rise of mark-up rate in the sectoral 2 on the equilibrium of capacity utilization. According to the relationship among m_1, m_2, p and θ_1, we find that θ_2 rises, m_1 remains unchanged, m_2 rises and p decreases. Therefore, when the mark-up rate of sector 2 (θ_2) increases, the intercept of equation (2.18) in the quadrant graph of (u_2, u_1) will increase. However, because the output of consumer goods sector is not used as intermediate input, there is no spillover effect, the increase is less than that of intercept in the other case. In addition, because the increase of θ_2 increases the price of consumer goods, leading to the “relative price decline” which depresses the investment of sectoral 1, while reduces the actual income of workers, the consumption expenditure of the whole society, so the rise of θ_2 makes the counterclockwise rotation of equation (2.20) larger than that of θ_1, and produces more products. The negative impact of capacity utilization rate is more obvious. Therefore, compared with the increase of θ_1, the rise of the θ_2 often has a negative impact on the capacity utilization rate. This makes it easier for the economy to have inadequate capacity utilization.

Therefore, the increase of the mark-up rate of sector 1 θ_1 and sector 2 θ_2 in the economy have “consumer demand effect" and “investment demand effect". The equilibrium of

TABLE 2. Three Expansion Regimes

Mark-up rate change	Impact on capacity utilization	Expansion regime	The conditions
	Rise in u_1, u_2	Profit Driven	$\alpha_1, \alpha_2, \gamma_1$ are large
A rise in θ_1	Rise in u_1,Fall in u_2	Hybrid	Intermediate parameters
	Fall in u_1, u_2	Wage Driven	$\alpha_1, \alpha_2, \gamma_1$ are small
	Rise in u_1, u_2	Profit Driven	$\alpha_1, \alpha_2, \gamma_1$ are larger
A rise in θ_1	Rise in u_1,Fall in u_2	Hybrid	Intermediate parameters
	Fall in u_1, u_2	Wage Driven	$\alpha_1, \alpha_2, \gamma_1$ are small

productivity utilization rate of the new two sectors $(u_1^{**}, u_2{**})$ will occur in three situations: $(u_1^{**} > u_1^*, u_2^{**} > u_2^*), (u_1^{**} > u_1^*, u_2^{**} < u_2^*), (u_1^{**} < u_1^*, u_2^{**} < u_2^*)$ because the production of the consumption goods sector is not used as intermediate input, there is no spillover effect for raise of the mark-up rate.

The new equilibrium will not occur $(u_1^{**} < u_1^*, u_2^{**} > u_2^*)$. As an investment goods sector, the expansion of each sector will inevitably lead to an increase in demand for investment products. Therefore, the increase of capacity utilization ratio of sector 1 is a prerequisite for the increase of capacity utilization ratio of sector 2. This also echoes the premise (necessary) condition that Marx put forward in his analysis of the expanded reproduction of capitalism: the surplus value of the means of production sector is greater than the invariable capital of the means of consumption sector $(I(v+m) > IIc)$;

In summary, when a rise in the mark-up rate in one sector increases (resp. decreases) the short-run equilibrium rates of capacity utilization in both sectors, the economy is called a profit-led (resp. wage-led) expansion regime. Moreover, when a rise in the markup rate in one sector increases the short-run equilibrium rate of capacity utilization in the investment goods sector but decreases that in the consumption goods sector, the economy is called a hybrid expansion regime. These three types of expansion regimes are summarized in Table (2).

4. **Long-Run Analysis**

In the analysis of the previous section, we regard k as a fixed value, and then analyze the growth mechanism of Marx-Kaleckian two sectors model. We get the relationship between the mark-up rate and the growth mechanism. We find that the profit-driven growth mechanism and the wage-driven growth mechanism analyzed by the traditional Kaleckian model are still valid, and there is a mixed growth mechanism. Fujita (2018) did not find the long-term equilibrium capital ratio k^*, and regarded the capital ratio k as a fixed value, so it was difficult to capture the relationship between the sector ratio k and the mark-up rate in capitalist economic growth. In this section, we will analyze the change of capital ratio k.

We first consider the equilibrium solution k^*. In capitalist society, the pursuit of profit is the basis for capitalist decision-making. If there is a gap between the profit rate of the two types of producers, capitalists have reason to change the use of capital in order to seek higher profit rate. In the long run, just as labor mobility tends to bring wages into equality, capital mobility will bring profit margins into equality, so that a equilibrium capital ratio k^* is required to meet the profit rate of the two sectors equally:

$$r_1 = r_2.$$

Decomposing the profit rate, we obtain the relationship between the profit share and capacity utilization

$$r_1 = \frac{R_1}{K_1} = \frac{R_1}{X_1}\frac{X_1}{K_1} = m_1 u_1, r_2 = \frac{R_2}{K_2} = \frac{R_2}{X_2}\frac{X_2}{K_2} = m_2 u_2.$$

In flow equilibrium, investment must be equal to savings, so the long-run investment function must be adjusted to make investment equal to savings. So the relationship between capacity utilization of the two sectors is determined by equation (2.20). Substituting equation (2.20)

$$m_1 = m_2 \frac{(1-m_1)(1-a_{11})pk + (1-s)(1-a_{11})m_1 pk}{1-(1-m_2)(1-a_{12}p)-(1-s)(1-a_{12})m_2}, \tag{4.1}$$

rearranging the equation (4.1), we obtain the equilibrium solution k^*:

$$k^* = \frac{m_1[1-(1-m_2)(1-a_{12}p)-(1-s)(1-a_{12})m_2]}{pm_2[(1-m_1)(1-a_{11})+(1-s)(1-a_{11})m_1]}. \tag{4.2}$$

We take the partial derivatives of k^* with respect to the profit share (m_i) and the relative price p:

$$\frac{\partial k^*}{\partial m_1} > 0,$$

$$\frac{\partial k^*}{\partial m_2} < 0,$$

$$\frac{dk^*}{dp} = \frac{\partial k^*}{\partial p} + \frac{\partial k^*}{\partial m_2}\frac{\partial m_2}{\partial p} > 0.$$

It can be seen that the equilibrium solution k^* is an increasing function of m_1 and a decreasing function of m_2. Intuitively, it can be understood that when the profit share of any sector increases, capital will flow to this sector.

We translate the relationship between sector 1 and sector 2 reflected in Table (1) into Marxian two sector reproduction schema. At this time, the price of commodities is essentially the production price, and the mark-up rate is expressed as the average profit rate. We simplify Table (1) into Table (3) to reflect the above relationship:

TABLE 3. Two Sector Reproduction Schema

	Constant Capital	Variable Capital	Surplus Value	Total Value
Sector I (Investment Goods)	$a_{11}p_1X_1$	Wb_1X_1	$\theta_1(a_{11}p_1 + Wb_1)X_1$	p_1X_1
Sector II (Consumption Goods)	$a_{12}p_1X_2$	Wb_2X_2	$\theta_2(a_{12}p_1 + Wb_2)X_2$	p_2X_2

According to Marx, in the long run, the profit rate of the two sectors will be equal, so that we obtain:

$$\theta_1 = \theta_2 = \theta^*$$

If the organic composition of capital are same in the two sectors, *i.e.*

$$\frac{a_{11}p}{b_1 w} = \frac{a_{12}p}{b_2 w},$$

we can obtain

$$p = \frac{b_1}{b_2}$$

$$k^* = \frac{b_2 s \theta^*}{b_1 \left[1 + (1 - s)\theta^*\right]}$$

We previously assumed that the short-run investment function was determined by equations (2.15) and (2.16), so we substituted equations (2.15) and (2.16) to the equation of capital ratio. We obtain:

$$\dot{k} = (g_1 - g_2)k,$$

where the dot over the variable represents its time derivative. In the long-run equilibrium where $\dot{k} = 0$ holds, means $g_1 = g_2$, we obtain the results from equation (2.23)- (2.24):

$$\frac{\partial g_1}{\partial k} < 0,$$
$$\frac{\partial g_2}{\partial k} > 0.$$

So the equilibrium point is globally asymptotic stable.

5. **Concluding Remarks**

Most previous Kaleckian research studied macrolevel functional income distribution and macroeconomic performance. However, in real economies, different industries have different market structures, and accordingly how sectoral income distribution/degree of monopoly affects sectoral as well as macroeconomic performance should be considered. We constructed a two-sector model and investigated how a sectoral mark-up rate change affects sector-/macro-level rates of capacity utilization and capital accumulation.

Firstly, the basic judgements of Post-Keynesian school, which includes paradox of thrift, economic fluctuction driven by animal spirit, profit driven expanision and wage driven expanision, are still valid in this model. Secondly, we found that the two-sector Marx-Kaleckian model has a hydrid expanision that is each sector's rate of capacity utilization moving in the opposite direction. Thirdly, under assumption that the profit rate of the two sectors are equal and the organic composition of capital are the same, we caculate the equilibrium ratio of capital stocks and prove global asymptotic stablility.

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