Post Keynesian Review
Vol. 6, No. 1, Pages 23–38 (the 1st March, 2019)
ISSN 2187-4875

An Empirical Study of Japan's Economy with Hoover's Method: Harrodian Instability and the Gap between Natural and Warranted Rates of Growth

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Abstract. Hoover(2008) developed an empirical test based on the implied reasoning of Harrod(1939)'s dynamic theory: gross domestic product (GDP) gaps should be inversely related to the difference between the natural and proper warranted growth rates. (Gaps refer to the percentage by which actual output falls short of the potential output each period.) I call this hypothesis the Hoover curve. Hoover derived a downward-sloping regression line of the U.S. economy for 1930-2005. In this paper, I discuss the Harrodian dynamics of economic growth. Harrod insisted on the principle of instability for trade cycle analysis. He thus diagnosed the long-run trend of an economy. It is deflationary and stagnated in cases where the warranted rate is greater than the natural rate of growth. Otherwise it is inflationary boom. I show a Hoover curve with structural change in Japan's economy, 1957-2014, and briefly describe the economic trends that explain those changes.

Keywords: Harrodian instability, warranted rate of growth, natural rate of growth, GDP gap, structural change.

1. Introduction

Harrod is well known for being a pioneer in modern economic growth theory. However, his theory has been discarded by the neoclassical school in favour of Solow(1956)'s thinking. Solow argued that Harrod's knife-edge equilibrium growth disappears once a fixed proportions production function was changed to a smooth neoclassical one.

Hoover(2008) discussed why Harrod's growth theory was discarded. He examined Harrod's 1939 article and noted that Harrodian instability did not depend on a fixed-proportions production function and unlike Harrod, Solow assumed that ex ante savings and investment were always equal to ex post savings and investment.[1] He argued that Harrod conjectured that the relationship of the warranted (proper) and natural rate of growth would determine the

The first draft received: the 31st October, 2018; The final accepted: the 20th March, 2019.

[1]See Bioanosky amd Hoover(2009), Hagemann(2009), and Halsmayer and Hoover(2015) for recent discussion of Harrod-Domar and Solow.

likelihood of an economy operating below full employment for any length of time.[2] Furthermore, he provided an empirical test regarding an implication from Harrod's dynamic theory. More specifically, the gross domestic product (GDP) gap should be inversely related to the difference between the natural and proper warranted rate of growth. I call this hypothesis the Hoover curve. He obtained a downward-sloping regression line for the case of the U.S. economy during 1930-2005.

In section 2 of this paper, I discuss Harrodian dynamics of economic growth with respect to the growth rate, G, and the warranted rate, G_w, *i.e.* the instability principle for trade-cycle analysis, and provide a formal formulation of the Harrodian instability. In section 3, I argue that a problem of a discrepancy between the natural rate, G_n, and the proper warranted rate, G_{pw}, has to do with a trend or structural analysis of a dynamic economy. It illustrates the dynamics of the economy in the two cases of difference of the natural and proper warranted rate of growth. In section 4, the Hoover curve is discussed in relation to the U.S. economy. In section 5, I show a Hoover curve with a structural change in the case of Japan's economy during 1957-2014. In section 6, I briefly discuss trends and changes in Japan's economy. Finally, in section 7, I provide conclusions.

2. **Harrodian Dynamics 1: G and G_w**

Harrod(1939) stated that a dynamic theory is 'thinking in terms of trends of increase' (p.15), putting the rate of growth of income (or output) as G, and he defined the word 'dynamic as referring to propositions in which a rate of growth appears as an unknown variable' (p.17).

Harrod defined the warranted rate of growth, G_w, as follows: 'The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount. Or to state the matter others, it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth' (p.16). He defined the following equation as the fundamental equation:

$$G_w = s/C, \tag{2.1}$$

where s is the savings rate and C is the value of the capital required for the production of a unit increment of output. G_w is 'the value of which is determined by certain 'fundamental conditions', namely, the propensity to save and the state of technology, *etc.* (p.17). 'This [s] may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc.' (p.16). 'It [C] may be expected to vary as income grows and in a different phase of trade cycle; it may be somewhat dependent on the rate of interest' (p.17).

He called the following equation a truism:

$$G = s/C_p. \tag{2.2}$$

[2] Hoover(2008, pp.19-20)

'It is a truism, depending on the proposition that actual saving in a period $\cdots$ is equal to the additional to the capital stock. Total saving is equal to sx_0. The addition to the capital stock is equal to $C_p(x_1 - x_0)$. This follows from the definition of C_p, G is the rate of increase in total output which actually occurs; C_p is the increment of the stock of capital divided by the increment of total output which actually occurs' (p.18).

He proceeded with the instability principle as follows: 'Now suppose that there is a departure from the warranted rate of growth. Suppose excessive output, so that G exceeds G_w. The consequence will be that C_p, the actual increase of capital goods per unit increment of output, falls below C, that which is desired. There will be, in fact, an undue depletion of stock or shortage of equipment, and the system will be stimulated to further expansion. G, instead of returning to G_w, will move farther from it in an upward direction, and farther$\cdots$ Similarly, if G falls below G_w, there will be a redundance of capital goods, and a depression influence will be exerted; this will cause a further divergence and a still stronger depression influence; and so on$\cdots$ A departure from equilibrium, instead of being self-righting, will be self-aggravating. G_w presents a moving equilibrium, but a highly unstable. Of interest this for trade-cycle analysis' (p.22).

Harrod's dynamics of the growth rate can be briefly described by the following:

$$dG/dt = \alpha(G - G_w), \tag{2.3}$$

where α is a positive constant and t is time. If G_w is constant, then the stationary point ($G = G_w$) is unstable. Equation (2.3) is unsuitable for the long-term, but is suitable for trade-cycle analysis; s and C are variables that depend on the phase of trade cycles and the interest rate. Therefore, the value for G_w changes during a trade cycle. Equation (2.3) describes a possible turning point when G is equal to G_w.[3] Harrod did not provide a complete dynamic system for the economy. He just proposed an idea for more dynamic thinking.

Okishio(1964) provided a formal proof of the instability of Harrod's model with attention to the capital utilisation rate and the investment function, having a fixed coefficient and a flexible production function.[4] His system with a fixed coefficient production is shown as follows:

$$sY = I, \tag{2.4}$$

[3]Harrod (1973, chapter 3) discussed lower and upper turning points in detail.

[4]In the case of production function, his system is as follows:

$$\sigma = f(n), f' > 0, f'' < 0, \tag{$a-1$}$$

$$f'(n) = w, \tag{$a-2$}$$

$$\delta = h(r), h' > 0, \tag{$a-3$}$$

$$r = \delta\sigma - w\eta n, \tag{$a-4$}$$

$$\eta = \eta(\delta), \eta' > 0, \eta(1) = 1 \tag{$a-5$}$$

$$\beta r = g, \tag{$a-6$}$$

$$dg/dt = \alpha(\delta_1), \tag{$a-7$}$$

(2.5) $$dK/dt = I,$$

(2.6) $$dg/dt = \alpha(\delta - 1),$$

(2.7) $$\delta \equiv Y/\sigma K,$$

(2.8) $$g \equiv I/K,$$

where $\equiv$: definition, Y: output, I: investment demand, s: savings rate (constant,0< s<1), δ: rate of capital utilisation, σ: normal output-capital (constant), K: capital stock, g: growth rate of capital, α: positive constant. Equation(2.4) is an equilibrium condition of the goods market. Output is determined by a principle of effective demand. Equation (2.5) means that capital accumulation is equal to investment demand. Equation (2.6) is an investment function of the Harrod-Okishio type. The utilisation rate indicates the current level of capital shortage or excess capacity. If the utilisation rate is greater than 1, *i.e.* a capital shortage exists, then firms will increase the capital growth rate by more than the previous year's level and *vice versa*. Equation (2.7) gives the definition of the utilisation rate of capital equipment. Equation (2.8) defines g.

Equations (2.4)-(2.8) are reduced to the following equation.

(2.9) $$dg/dt = \alpha(g/s\sigma - 1).$$

The stationary point ($g^* = s\sigma$) is clearly unstable and has a 'knife-edge' property.[5] Equation (2.9) can be translated in a Keynesian style. Let

$$y = Y/K,$$

and from (4) and (8) we get:

$$y = \frac{Y}{I}\frac{I}{K} = \frac{1}{s}g.$$

The time-derivative of the above equation and (9) yields the following equation:

(2.10) $$dy/dt = (\alpha/s\sigma)(y - \sigma).$$

If the actual growth rate exceeds the warranted rate, then $y > \sigma$ and y increases, leading to even more excess demand relative to capital. The economy moves into an explosive excess

where n: normal labour-capital ratio, $f(n)$: production function, w: real wage rate, r: rate of profits, $h(r)$: utilization rate function of rate profits. η: actual - normal ratio of employment, $\eta(\delta)$: employment function, β: savings rate from profits. (a-2) is a condition of profit maximization on choice of techniques. (a-3) is an another expression of a short run supply function. (a-4) is a definition of the profit rate. (a-5) is a technical condition of actual employment and output. (a-6) is a condition of saving-investment equality under the assumption that capitalists save some of their profits and that workers consume all their income. (a-7) is an investment function. The above 7 equations determine the 7 variables, $\sigma, n, \delta, r, \eta, w$ and g. Dynamics of the system can be reduced to one differential equation of g:

$$dg/dt = \alpha(h(g/\beta) - 1), \qquad (a-8),$$

The stationary point is clearly unstable because $h' > 0$.

[5]Yoshida(1999) considered the instability of the Harrodian model with a flexible production function.

demand trajectory in which despite rising investment a growing shortage of capital stock relative to demand persists. If the actual growth rate is less than the warranted rate, then $y < \sigma$ and y falls, leading to an even larger excess capacity. The growth rate declines and, despite the slower growth (or absolute decline) of investment, a growing surplus of capital stock relative to demand exists.

3. **Harrodian Dynamics 2: G_N and G_{PW}**

Harrod (1939) proposed the concept of the natural rate of growth: 'This[the natural rate of growth] is the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense'(p. 30). He introduced the concept of the proper warranted rate of growth: 'Indeed, there is no unique warranted rate; the value of warranted rate depends on the phase of the trade cycle and the level of activity. Consideration may be given to that warranted rate which would obtain in conditions of full employment; this may be regarded as the warranted rate "proper" to the economy'(p. 30).[6]

Then, he put forward his long-run or trend analysis: 'If the proper warranted rate is above this[the natural rate], there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment. The warranted rate is dragged down by depression; it may be twisted upwards by an inflation of prices and profits. If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profits booms' (p. 30).

Now, let us assume that we are proceeding on the full employment growth path and that, at one time, the natural rate of growth is less than the proper warranted rate of growth perhaps because of an increase in the savings rate. As such, a decrease in consumption demand occurs and the actual rate of growth will decrease because the actual rate (= the natural rate) is less than the warranted rate. Underemployment now exits for several periods. The growth rate decreases as long as it is smaller than the warranted rate.

Harrod's reasoning guiding his long-run analysis is based on the short-run instability principle. Even in the long-run, he does not postulate full employment of labour and capital unlike Solow and neoclassical growth theory do.[7] The problem of discrepancies between the natural and proper warranted rates of growth affects long-run trends in the economy. In contrast,

[6]The 'proper' warranted rate of growth term disappeared in Harrod(1948; 1973). Harrod (1973, p. 36) made a distinction between the 'normal' and 'special' warranted rate. The normal rate is the initial warranted rate as pertaining to a steady advance and the special rate changes under the influence of a boom or slump. Harrod (1973, chapter 7) discussed some problems and conflicts that arose from the discrepancy of the normal warranted and natural rate of growth.

[7]Nikaido(1975; 1980) discussed the instability of the equilibrium growth path with the Harrodian investment function under a neoclassical production function. Fanti & Manfredi(2009), Sportelli(2000) and Yoshida(2007) are recent elaborations of the Harrodian dynamic models. Hein *et al*(2011) provides a clear explanation of

the neoclassical school presumes a full employment economy and considers the stability of a steady growth economy.

This discrepancy problem can be formally analysed as follows:

$$N = nY, \tag{3.1}$$

$$dL/dt = \lambda L, \tag{3.2}$$

$$e \equiv N/L, \tag{3.3}$$

where N: employment, n: labour coefficient (constant), L: labour supply, λ: growth rate of labour force (constant), e: employment rate. Equation (3.1) indicates that employment is determined by output. Equation (3.2) means that labour supply increases at the constant growth rate. Equation (3.3) is the definition of the employment rate. We assume and analyse the underemployment economy.[8] Let

$$k = K/L, \tag{3.4}$$

and (3.3) yields, from (2.4),(2.7),(2.8) and (3.1):

$$e = nkg/s. \tag{3.5}$$

The time-differentiation of Equation (3.4) yields

$$d \log k/dt = g - \lambda. \tag{3.6}$$

We shall focus on the movement of e and g. From (2.9), the time-differentiation of (3.5) , and (3.6), we get the following equation:

$$d \log e/dt = (g - \lambda) + \alpha(\frac{1}{s\sigma} - \frac{1}{g}) \equiv E(g). \tag{3.7}$$

$E(g)$ has the following properties:

$$E(0) = -\infty, E(s\sigma) = s\sigma - \lambda, E(\lambda) = \alpha(\frac{1}{s\sigma} - \frac{1}{g}),$$

$$dE(g)/dg > 0 \quad for \quad g > 0.$$

We denote g^e as a value of g such that $E(g^e) = 0$.

Then

$$\lambda < g^e < s\sigma \quad if \quad \lambda < s\sigma,$$

Harrodian instability, comparing it to Keynesian instability. Moudud (2009) discussed the role of the fiscal and taxation policy of warranted growth. See Dalgaard and Erickson(2006) and Shaikh(2009) on policy issues.

[8]Niisato(2016) analyses the case of full employment. Any path into full employment will be sooner or later gone out to the underemployment area.

$$\lambda > g^e > s\sigma \quad if \quad \lambda > s\sigma.$$

The system of Equations (2.9) and (3.7) describes the movement of g and e. Figures 1 and 2 illustrate the phase diagrams. Figure 1 is the case where the proper warranted rate, $s\sigma$, is greater than the natural rate of growth, λ. In Figure 1, the capital growth rate and the employment rate are declining even if the employment rate moves upwards for a while, after the capital growth rate is below the proper warranted rate. This indicates that a chronic tendency towards economic depressions exists. Figure 2 is the case where the proper warranted rate is less than the natural rate. In Figure 2, the capital growth rate and the employment rate are moving upwards even while the employment rate falls temporarily, after the capital growth rate has exceeded the warranted rate. This implies that a tendency for profit booms occurs.

4. **Hoover Curve of the U.S.**

Hoover(2008) took Harrod's analysis as an empirically testable proposition: 'Harrod's conjecture says that an economy in which the proper warranted rate of growth (G_{PW}) stands above the natural rate of growth will display'a chronic tendency to depression' and an economy in which the proper warranted rate stands below the natural rate may be frequently driven towards full employment' (p. 21). 'One implication of Harrod's hypothesis is that the output gap should be inversely related to the difference between the natural and the warranted rate ($G_N - G_{PW}$)' (p. 22).

Hoover constructed a potential output series (Y_t^{pot}) for the U. S. for 1929-2005. The time series was generated from a Cobb-Douglas production function with the available labour force and capital stock. The level of the total factor productivity is estimated to grow smoothly along the upper bounds of the actual total-factor productivity. From the time series for potential output and capital stock, estimates of capital-output ratio (C) and the net savings rate (s) can be constructed, eventually yielding an estimate of the proper warranted rate of growth rate for each period: $G_{PW,t} = s_t/C_t$. The natural rate (G_N) is just the rate of growth of potential output for each period. The output gap is simply the percentage by which actual output falls short of the potential output each period: $Gap_t = (Y_t^{\text{pot}} - Y_t)/Y_t^{\text{pot}}$.

With regression analysis, Hoover obtained a downward-sloping regression line:

(4.1)
$$Gap = -5.9(G_N - G_{PW}) + 16.1$$
$$R^2 = 0.40$$

His Figure 5[9] illustrates that 'this is indeed the case: when the U.S. economy has a proper warranted rate of growth that was low relative to the natural rate, it has operated near to full potential' (p. 22).

[9]Figure 5 in Hoover(2008) is reproduced in Appendix 1, as our Figure 1. Prof. Hoover added the years and lines on it and provided the Figure to the author.

5. Hoover Curve of Japan

I applied Hoover's method to Japan's economy. First, I constructed a potential output series for Japan for 1956 − 2014 (Appendix 2 has the data construction details), and then estimated the potential GDP. The GDP Gap and GN were estimated for 1957−2014. G_{PW} was estimated as a trend of the capital growth rate. A scatter diagram with connected lines and years is shown in Figure 3. Looking at the time configuration of Gap and the difference of G_N and G_{PW}, it is easily observed that there were two Hoover curves. Hence, some structural change must have occurred after 1970s. It is well known that in the early 1970s, the rapid growth period ended: therefore, 1970 was a turning point.[10]

Two estimations were obtained as follows:

For 1957 − 1972

(5.1)
$$Gap = -11.2518(G_N - G_P W) + 9.385251,$$
$$(-5.70369) \quad (3.8656061),$$
$$R^2 = 0.699133DW = 0.879934.$$

(5.2)
$$Gap = -4.21359(G_N - G_{PW}) - 2.27181,$$
$$(-16.3748) \quad (-2.11038),$$
$$R^2 = 0.867371DW = 1.285624.$$

The determinant coefficient R2 and the Durbin-Watson statistic were good and the estimated values were all sufficiently significant (at the 5% level). The two estimated lines are illustrated in Figure 3, implying that Japan's economy had structurally changed in the early 1970s.[11] If G_{PW} falls and the slope changes, then the estimation line also changes. The estimation moved to the left by 2.77% point in this case,[12] which implies that G_{PW} fell from 6.756% to 3.989%. The falter curve means the firms had a less-active investment behaviour.

The shift of the estimated line would reflect the decline in the proper warranted rate of growth, $G_{PW} = s/C$, with a smaller net savings rate, s, and/or larger capital-output ratio, C, in the long run. In an open economy, from saving and investment balances,[13] it follows that:

[10] See Nakamura (1995, chapter 6)and Yoshikawa (1995, pp. 25-6 and p. 38) on the end of rapid growth. See Boyer and Yamada(2000) and Okishio (1992, chapter 2) on Japan's economy from 1950s to 1990s.

[11] The Chow test yields effectively structural change in 1972.

[12] The Hoover curve $Gap = -a(G_N - G_{PW}) + b$ is shifted down if G_{PW} is smaller. Let the new Hoover curve be $Gap = -a'(G_N - (G_{PW} - c)) + b = -a'(G_N - G_{PW}) + b'$. Then, $-a'c + b = b'$. Therefore, from (5.1) and (5.2), the value of c is calculated as $c = (b - b')/a' = 2.7665\%$

[13] According to the national account identity, $Y = C + I + G + EX - IM. \Delta K = I - D = (Y - D - T - C) - G + T - (E - M) = S_p - (G - T) - (E - M).(I - D)/Y = s_p(Y - D - T)/Y - (G/Y - T/Y) - (EX/Y - IM/Y)$. This equation is (5.3).

(5.3) $$s = s_p(1 - d - t) - (g - t) - (x - m),$$

where s_p is private net savings rate of NDP, d is the depreciation rate of GDP, g is government expenditure rate of GDP, t is the tax rate, x is the export rate and m is the import rate. A decrease in s is caused by (1) a decline in s_p, (2) an increase in d, (3) an increase in t, (4) an increase in the full employment budget deficits rate, $g - t$, and (5) an increase in the foreign trade surplus rate, $x - m$. These changes in Japan's economy have already been pointed out by many scholars.[14]

6. **Trends and Changes in Japan's Economy**

Figures 4 and 5 illustrate GDP Gap and the gap between G_N and G_{PW}, respectively, in a time series form. By observing the ups and downs in these figures, the ensuring periods were easily distinguished and corresponded to the economic changes in Japan's economy:

- 1957-1972 rapid economic growth:
  A declining trend in the GDP Gap and the zero gap in 1970. An increasing trend in G_N. The difference between G_N and G_{PW} disappeared in 1970.
- 1972-1974 full employment and yen-evaluation:
  Almost zero GDP Gap (*i.e.* full employment). G_N exceeded G_{PW}.
- 1974-1986 oil shocks and stable growth:
  After a sudden increase in the GDP Gap, the ensuring gap was relatively constant. After the sudden fall of G_N below G_{PW}, there was a stable difference because of the falling G_N and a decline in the G_{PW}.
- 1986-1991 bubble economy:
  A diminishing GDP Gap. An increasing trend in G_N.
- 1991-2002 lost 10 years:
  An increasing GDP Gap and a peak in 2002. An enlarged difference between G_N and G_{PW} owing to a decline in G_N.
- 2002-2008 Koizumi structural reform:
  A diminishing but high level GDP Gap. A small recovery of the difference between G_N and G_{PW}.
- 2008-2014 Lehman shock and after:
  A sudden increase in the GDP Gap in 2009 is owing to the Lehman shock in September 2008. The Grate Earthquake of East Japan on 11 March in 2011 did not have a significant negative effect. In other words, there was no change in the long-term trend towards stagnation.

In Figure 5, the proper warranted rate of growth, G_{PW}, is a constant. The ups and downs in the graph reflect the movement of the natural rate, G_N. Broadly speaking, there was an

[14]See Uni(2000) for a trend of C, and Ihori(2007,p. 109) and Hiromatsu, *et al.* (1998, p.110) for full employment budget deficits.

increasing trend of the G_N until the early 1970s, a low level of G_N during the early 1980s and a further decreasing trend in G_N after the early 1990s. Supposing that a downward shift of G_{PW} occurred in the late 1970s and early 1980s, I conclude that the bubble economy during late 1980s appeared in the condition where G_N was greater than G_{PW}.[15] After then the great recession developed under conditions where G_N was lower than G_{PW}.

7. Conclusions

The Harrodian instability principle, which comes from a discrepancy between the actual and warranted rate of growth, is a short run tool used for trade-cycle analysis. A discrepancy problem between the natural and proper warranted rate concerns the long-run or trend analysis and is related to the structure of the economy. A Hoover curve is a testable proposition of the long run trend of the economy.

In this paper, I showed a Hoover curve (with a structural change occurring in 1972) in Japan's economy during 1957-2014. This implied an increasing trend in the natural rate of growth, G_N, until the early 1970s, a low level of G_N during the early 1980s, a decreasing trend in G_N after the early 1990s and a shift down in the proper warranted rate of growth, G_{PW}, after the late 1970s. I also briefly discussed some trends and changes in Japan's economy using a Hoover curve since the 1950s. I concluded that the bubble economy during the late 1980s appeared under conditions where G_N was greater than G_{PW}. After that the great recession developed under conditions where G_N was lower than G_{PW}. And these circumstances still remain today.

Needless to say, further research should analyse Japan's economy in more detail. However, it is plausible that Harrodian dynamic theory now has greater empirical evidence to back it, and it has become perhaps another effective way to analyse economic growth and trade cycles.

Appendix A. Hoover Curve of the U.S.

Source: Hoover, Kevin D. (2008).

Appendix B. Japanese Data Construction

1) Data Source

ESRI(economic and Social Research Institute) Website

(http://www.esri.cao.go.jp/jp/sna/menu.html), accessed on 22 April 2016.

Gross domestic product (NY)

Real gross domestic product (Y), fixed-based approach on 2005

Compensation of Employees

Private capital stock (K)

[15]Considering a shift down of G_{PW} by 2.76% point after 1972, the horizontal line moves down by the same degree as Figure 5.

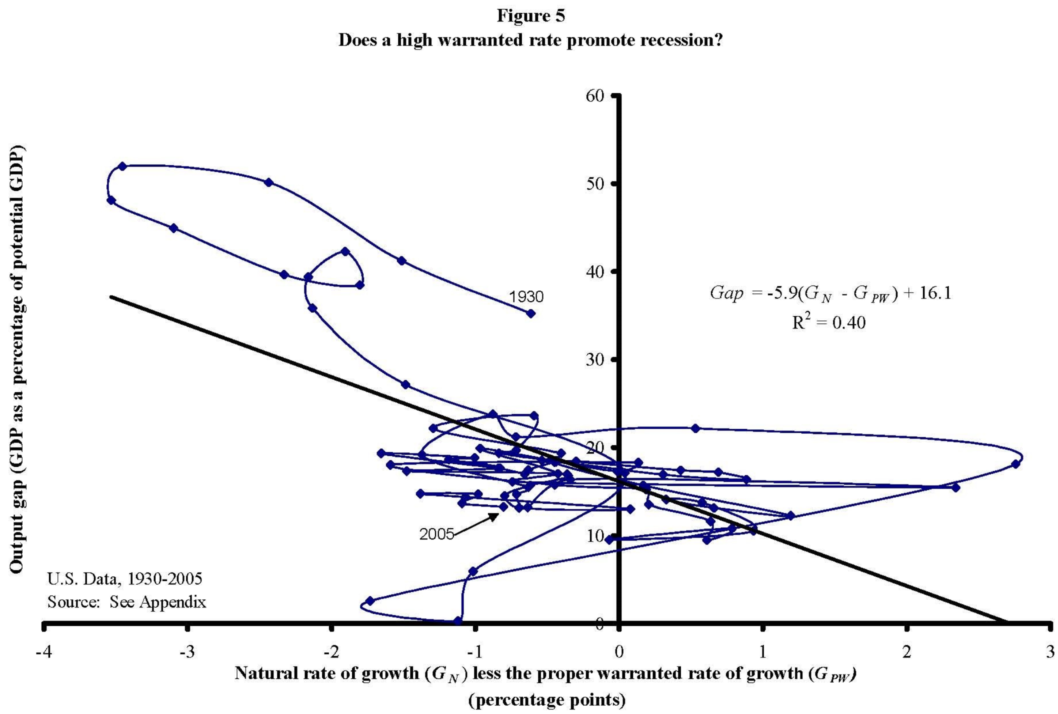

FIGURE 1. The Original Image by Professor Hoover

National Account for 2016 (benchmark year=2005, 93SNA):

real GDP, nominal GDP and Compensation of employees for 1994-2014

National Account for 2000

real GDP, nominal GDP and Compensation of employees for 1955-1998

a series of real GDP(benchmark year=2000) is made from connecting two series at the 1980.

Ministry of Internal Affairs and Communications Website (http://www.stat.gp.jp/data/roudou/index.htm), accessed on 22 April 2016.

Lobour Force (LF): Source: Labour Force Survey

2) Construction of the Output Gap.

Labor share in output(α): $\alpha_t = \frac{\text{compensation of employees}}{NY_t - \text{personal firm income}}$.

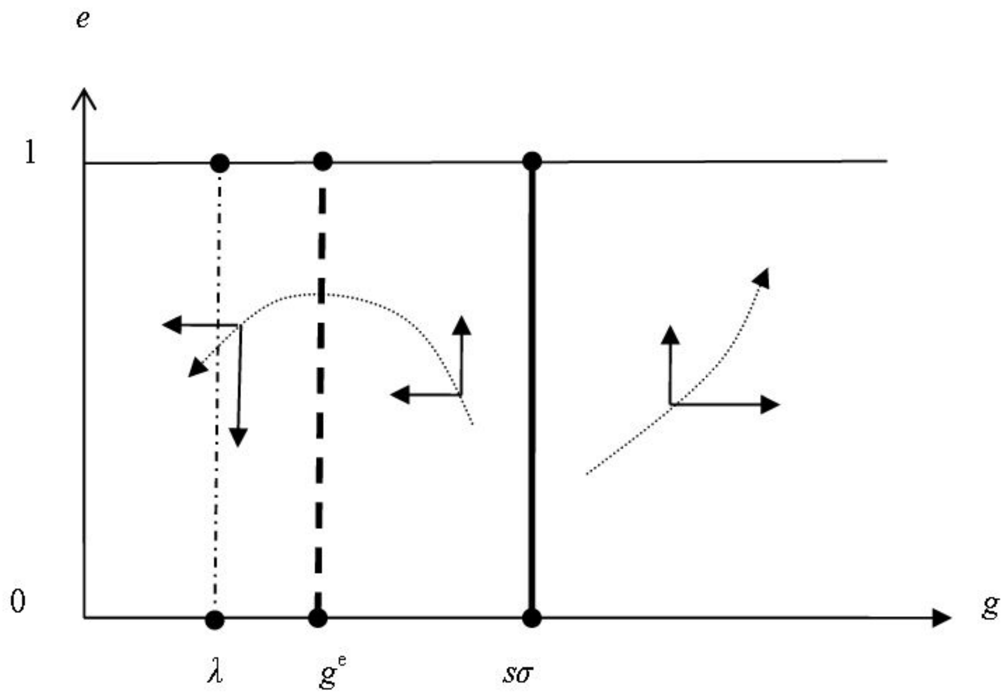

FIGURE 2. $g - e$ Phase Diagram in Case 1 ($s\sigma > \lambda$)

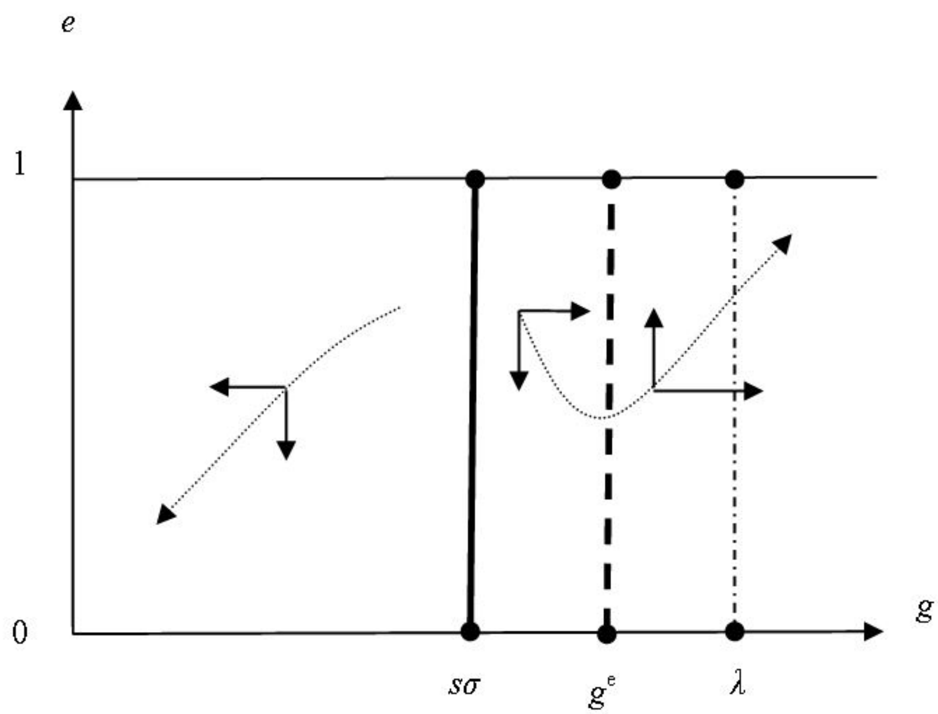

FIGURE 3. $g - e$ Phase Diagram in Case 2 ($s\sigma > \lambda$)

Personal firm income includes personal income in agriculture and fisheries.

$$\alpha = \text{the mean of } \alpha_t = 0.5731.$$

Total-factor productivity (A): $A_t = \frac{Y_t}{LF_t^{\alpha} K_{t-1}^{(1-\alpha)}}$, where K_{t-1} means the value of capital at the end of the year $t-1$ (=the beginning of the year t). In the above definition, the use of the labor force (LF) rather than employment has the effect of incorporating the "inefficiency" of the unemployment into A.

Full employment total-factor productivity (A^{full}): an exponential trend is fitted to A by ordinal least squares:

$$\ln \hat{A}_t = 0.0047 \text{ time } - 7.2544. \quad \ln A_t^{\text{full}} = \ln A_t + 7.0751.$$

The additional constant has the effect of shifting the whole path of $\hat{A}_t$ so that it forms the outer envelope of the A_t.

Potential GDP (Y_t^{pot}) : $Y_t^{\text{pot}} = A_t^{\text{full}} LF_t^{\alpha} K_{t-1}^{(1-\alpha)}$.

GDP Gap (Gap_t) : $Gap_t = \frac{Y_t^{\text{pot}} - Y_t}{Y_t^{\text{pot}}}$.

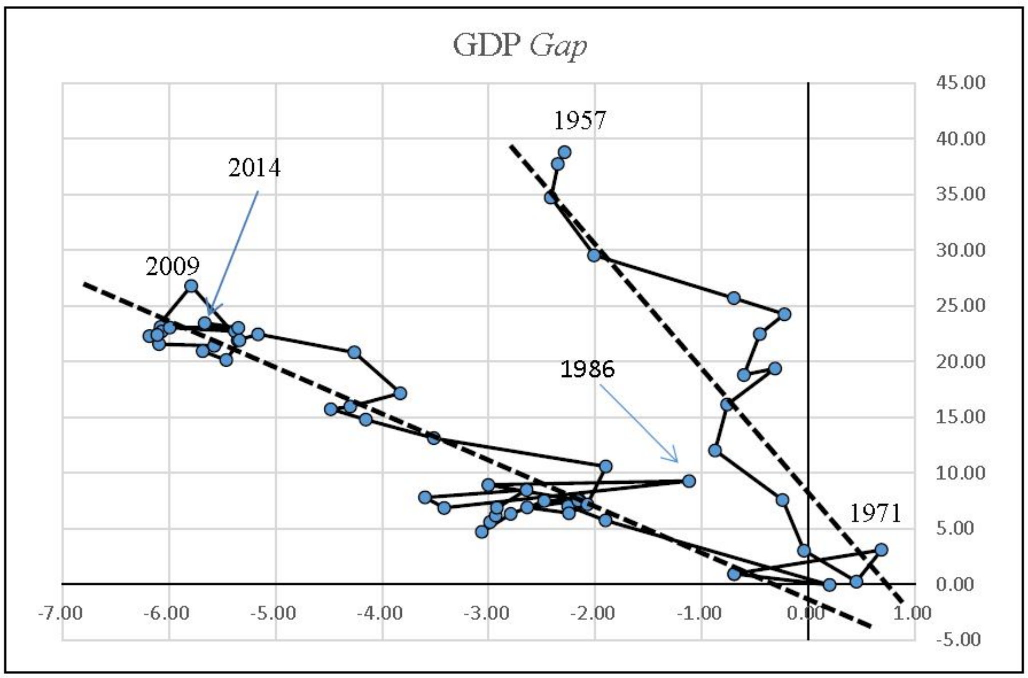

FIGURE 4. Shift of Hoover Curve, Japan:1957-1972, 1972-2014

3) Construction of the Natural Rate of growth.

Natural Rate of Growth (G_N) : $G_N = \frac{Y_t^{\text{pot}} - Y_{t-1}^{\text{pot}}}{Y_{t-1}^{\text{pot}}}$.

4) Construction of the Proper Warranted Rate of Growth.

Trend capital ($\hat{K}$): An exponential trend is fitted to K by ordinary least square:

$$\ln \hat{K}_t = 0.06537 \text{ time } - 116.96.$$

Capital-output Ratio (C): $C_t = \frac{\hat{K}_{t-1}}{Y_t^{\text{pot}}}$.

Net Savings Rate (s): $s_t = \frac{\hat{K}_t - \hat{K}_{t-1}}{Y_t^{\text{pot}}}$.

Proper Warranted Rate of Growth (G_{PW}) : $G_{PW} = \frac{s_t}{C_t} = \frac{\hat{K}_t - \hat{K}_{t-1}}{\hat{K}_{t-1}} = 6.756\%$.

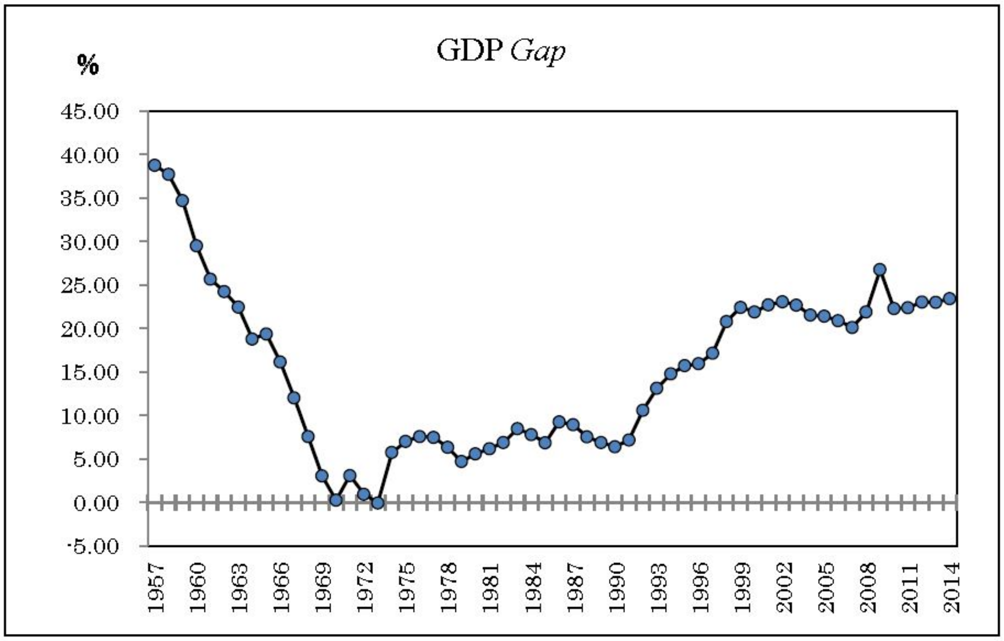

FIGURE 5. GDP Gap 1957-2014

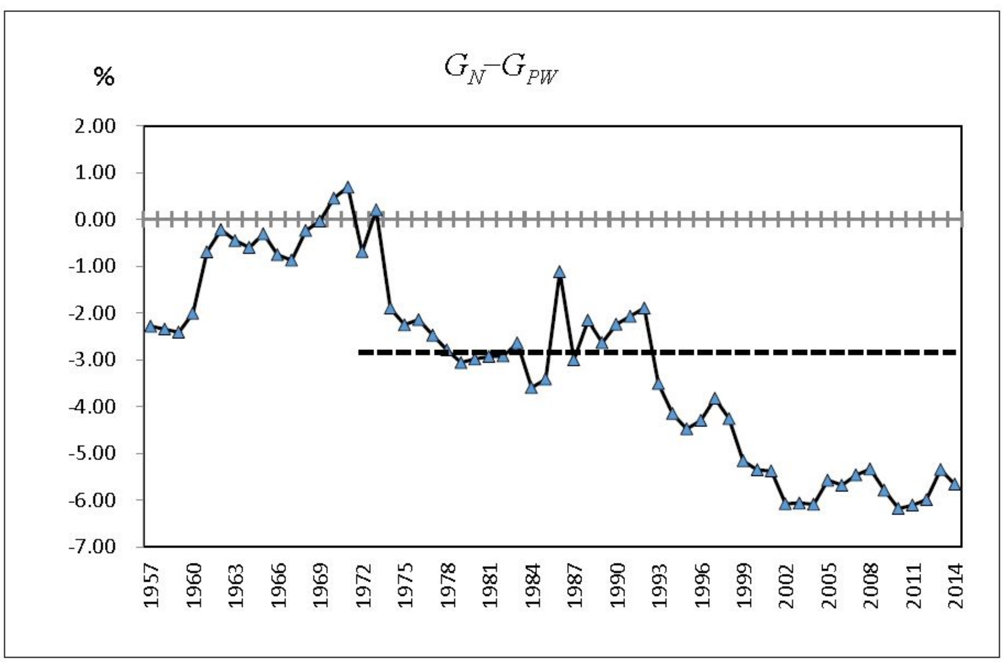

FIGURE 6. Growth Rate Gap 1957-2014

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Acknowledgements: I greatly thank Professor Kevin Hoover for providing the figure in Appendix 1 which stimulated this paper. I also thank Heinz Kurz, Geoffery Hourcort, Kazuo Ogawa, Masaaki Tokuda, Takeshi Nakatani, Neri Salvadori and Robert Rowthorn for their helpful comments. Usual caveats apply.