

Two Notes on Dynamics of Fixed Capital

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ABSTRACT. This notes present two propositions on fixed capital. The first one concerns extension of the so-called Cambridge-equation. We start from the system of commodity production, in which aged fixed capital is jointly produced. By eliminating aged fixed capital, we can establish equilibria of production prices and the uniform growth path. With respect to gross profit, *i.e.* the internal reserve, and the accumulation of capital from gross profit, we establish the Cambridge equation. The second concerns accelerated depreciation of fixed capital. If depreciation of fixed capital is made in a period shorter than its durability and depreciation is reinvested for fixed capital at once, then the oscillation of capacity of the production process concerned will have a larger spike, and thus the whole economic process will be disturbed harder than otherwise.

1. Introduction

Fixed capital is the central core of materialistic capital. Hence, the centre of capital accumulation is the accumulation of fixed capital.

One of most important features of fixed capital is that it resides in the production process for more than one cycle of reproduction. Hence, evaluation of fixed capital needs a special consideration in accounting. That is, amortisation of fixed capital should be made.

Amortisation is a procedure to calculate a part of value of advanced fixed capital which should be recovered in the current cycle of production. In this sense, amortisation is a part of cost of production, first of all.

As opposed to liquid cost, however, the amount of depreciation is not usually kept at the hands of producers until fixed capital should be replaced. Hence, there is a possibility that the amount of depreciation can be reinvested for additional units

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of fixed capital. In this case, the amount of depreciation plays a role, as if it were a part of the source for investment.

The plan of the article is as follows. Section 2 is for the equilibrium analysis of the economy *à la* Marx-Sraffa. Section 3 will focus on depreciation of fixed capital and its reinvestment, which has been known as the Marx-Engels effect or the Ruchti-Lohmann effect. It is confirmed that reinvestment gives a not small impact on the economy. In Section 4, further problems of accelerated depreciation, will be taken up. Section 5 will give short, tentative conclusions and future problems.

2. Marx-Sraffa's Price Basis

Price and quantity equilibria of joint production systems will be summarised here.

A general joint-production system is not discussed, however. In what follows, it is *assumed* that aged fixed capital is jointly produced; there is no process that produces only aged fixed capital.¹

Let B and M stand for the output and the input matrices, respectively. M includes indirect inputs of wage goods. Assume that B and M are of the same dimension, say $m \times n$; the economy consists of m types of commodities and n processes. Let p stand for an m -dim price vector, and the uniform profit rate equilibrium is expressed by p such that

$$pB = \lambda pM,$$

where λ denotes the profit factor.

Sraffa(1960) showed that the system of equations in the above can be reduced to a system with only brand-new commodities. Okishio-Nakatani(1975) applied the reduction procedure to the Marxian system with *ex ante* wages. Asada(1982) named these models as SON. A formal summary of this reduction is given in Li-Fujimori(2013).

We consider here a small scale, simplified linear model of this kind. Put

$$M = \begin{pmatrix} k_1 & 0 & 0 & k_2 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 \\ 0 & 0 & k_1 & 0 & 0 & k_2 \\ fl_1 & fl_1 & fl_1 & fl_2 & fl_2 & fl_2 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ k_1 & 0 & 0 & k_2 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix},$$

¹ This kind of economies was named as the *plain economy* by Shiozawa. See Fujimori(1982, p.36.)

and the full price vector is expressed by: $p = (p_1 \ p_1^1 \ p_1^2 \ p_2)$, where p_1^i indicates fixed capital of age i .

In the reduced system, to which only brand-new commodities belong, the system will be represented by a set of the following:

$$K = \begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix}, \widehat{\psi}(r, \tau) = \begin{pmatrix} \psi(r, \tau) & 0 \\ 0 & 1 \end{pmatrix}, F = \begin{pmatrix} 0 \\ f \end{pmatrix}, L = \begin{pmatrix} l_1 & l_2 \end{pmatrix}$$

with the *rate of depreciation* $\psi(r, \tau) = \frac{1}{1 + (1+r) + \dots + (1+r)^{\tau-1}}$, $\tau = 3$, for durability τ . One obtains

$$\bar{p} = \bar{p}(rI + \widehat{\psi}(r, \tau))K + \bar{p}(1+r)FL$$

for an equilibrium price vector $\bar{p} = (p_1 \ p_2)$.

3. Pasinetti's Formula

If fixed capital is applied in the production process, the Pasinetti Formula (Cf. Pasinetti, 1962) should be reformulated. It is Koshimura(1968) that first tried to reestablish the Pasinetti formula with fixed capital. Koshimura built a model in the value terms, so that his extension turned out to be a rather complicated one.

Li(2012) extended the Pasinetti formula to the case with fixed capital.

On the quantity side, one has

$$(3.1) \quad Bx = (1+g)Mx + u,$$

where u stands for consumption. A similar reduction of the system with aged fixed capital to the system with only brand-new commodities is possible.

The sum of brand-new commodities produced throughout the economy is given by the following:

$$(3.2) \quad q_i = \sum_{h=1}^3 x_i^h.$$

If the economy is in the balanced growth state, one has

$$(3.3) \quad k_i x_i^3 = \frac{1}{1+g} k_i x_i^2 = \frac{1}{(1+g)^2} k_i x_i^1.$$

$$(3.4) \quad q = \left((\widehat{\psi}(g, \tau) + gI)K + (1+g)FL \right) q + C,$$

TABLE 1. Input-Output Relationships

process	inputs			outputs		
	1	2	3	1	2	3
t	0	$k_1 x_1^1$		x_1^1		
	1			$k_1 x_1^1$		
	2					
$t + 1$	0	$k_1 x_1^2$		x_1^2	x_1^1	
	1		$k_1 x_1^1$	$k_1 x_1^2$		
	2				$k_1 x_1^1$	
$t + 2$	0	$k_1 x_1^3$		x_1^3	x_1^2	x_1^1
	1		$k_1 x_1^2$	$k_1 x_1^3$		
	2		$k_1 x_1^1$		$k_1 x_1^2$	

$$q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ \bar{C} \end{pmatrix}, \hat{\psi}(g, \tau) = \begin{pmatrix} \psi(g, \tau) & 0 \\ 0 & 1 \end{pmatrix}.$$

In the brand-new commodity world, one has

$$(3.5) \quad \bar{p}q = \bar{p}\hat{\psi}(r, \tau)Kq + r\bar{p}Kq + (1+r)\bar{p}FLq,$$

$$(3.6) \quad \bar{p}q = \bar{p}\hat{\psi}(g, \tau)Kq + g\bar{p}Kq + (1+g)\bar{p}FLq + \bar{p}C,$$

so that

$$(3.7) \quad r\bar{p}(K + FL)q + \bar{p}\hat{\psi}(r, \tau)Kq = g\bar{p}(K + FL)q + \bar{p}\hat{\psi}(g, \tau)Kq + \bar{p}C.$$

This implies that *gross profit* or *internal reserve* is expressed by

$$\text{net profit} + \text{depreciation} = \text{net investment} + \text{replacement} + \text{consumption}.$$

Hence, we define

$$\begin{aligned} \text{gross profit rate} &= \frac{\text{gross profit}}{\text{initial capital}}, \\ \text{rate of accumulation} &= \frac{\text{gross investment}}{\text{gross profit}}. \end{aligned}$$

Let $S(t)$ denote the nominal amount of fixed capital in period t . One has

$$(3.8) \quad \Delta S(t) = S(t+1) - S(t) = gS(t).$$

This can be rewritten as

$$(3.9) \quad \Delta S(t) = k_1 x_1^1 + k_2 x_2^1 + g(k_1 x_1^1 + k_2 x_2^1)$$

$$(3.10) \quad = \frac{g(1+g)^3}{(1+g)^3 - 1} (k_1 q_1 + k_2 q_2).$$

From (3.8),

$$S(t) = \frac{1}{g} \Delta S(t) = \frac{(1+g)^3}{(1+g)^3 - 1} (k_1 q_1 + k_2 q_2).$$

In the production-price term, one has

$$\frac{(1+g)^3}{(1+g)^3 - 1} (p_1 k_1 q_1 + p_1 k_2 q_2) = \frac{1}{g} (g \bar{p} K q + \psi(g, 3) \bar{p} K q).$$

Hence,

$$(3.11) \quad r = \frac{r \bar{p} K q + \psi(r, 3) \bar{p} K q}{\frac{1}{g} (g \bar{p} K q + \psi(g, 3) \bar{p} K q)},$$

$$(3.12) \quad \alpha = \frac{g \bar{p} K q + \psi(g, 3) \bar{p} K q}{r \bar{p} K q + \psi(r, 3) \bar{p} K q}.$$

Therefore,

$$(3.13) \quad \alpha \times r = \frac{g \bar{p} K q + \psi(g, 3) \bar{p} K q}{r \bar{p} K q + \psi(r, 3) \bar{p} K q} \times \frac{r \bar{p} K q + \psi(r, 3) \bar{p} K q}{\frac{1}{g} (g \bar{p} K q + \psi(g, 3) \bar{p} K q)} = g.$$

Thus, Pasinetti's formula holds in this case, if depreciation is evaluated by the so-called pension method.

Note that the above formula does not hold for the case of $r = 0$.

One sees that depreciation of fixed capital is a part of cost of production on one side, but on the other side it is a source for capital accumulation and appears as if it were a part of profit. Depreciation of fixed capital has two faces.

3.1. A small note on technologies with greater k . By way of excursion, we examine the sensisivity of gross profit to durability and the magnitude of k .

If we consider a point of equilibrium, it is clear that an increase in the input coefficient k will decrease the rate of profit.

In fact, this can be illustrated as follows. Suppose the simplest case that there is only one tye of brandnew commodities, that is, the other remaining commodities are all aged fixed capital. Assume that there is only one type of fixed capital of zero-age.

The model is characterised by the output- and input-matrices, such as

$$B = \begin{pmatrix} 1 & 1 & 1 \\ k & 0 & 0 \\ 0 & k & 0 \end{pmatrix}, \quad A = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix},$$

respectively. Since A and B are square matrices, one can compute the eigensystem of BA^{-1} . The reciprocal of eigenvalues, except zeros, gives the spectrum of $pB = \lambda pA$. It is evident that an increase in k will decrease λ with a constant level of the real wage rate, so that a greater fixed capital coefficient will pull down the rate of profit: $\frac{d\lambda}{dk} < 0$.

On the other hand, investment in heavier fixed capital will bring back a greater amount of depreciation, so that a greater amount of gross profit will be gained by capital-deeping investment.

In capitalists' economies, capitalists will avoid employing less efficient system of technologies. In the class struggle situation, however, in which there is a possibility to decrease the real wage rate or labour inputs by further investment in fixed capital, capitalists will resort to capital-deepening technologies with greater gross profit and a larger k in the short-run.

In fact, put $p = 1$ and $q = 1$, and the internal reserve $z = r(k + \omega l) + \varphi(r)k$. We have $dz = dr((1 + d\varphi)k + \omega l) + (r + \varphi(r))dk + rld\omega + r\omega dl$. Hence, there is a possible combination of $dz > 0$, $dk > 0$, $dr \geq 0$. (ω denotes wages.)

4. Marx-Engels-Ruchti-Lohmann-Kalecki-Steindl Effect

4.1. Introduction. Before discussing in depth the above two-sidedness of depreciation of fixed capital, we take up reinvestment of depreciation of fixed capital.

In their correspondence Marx and Engles discussed whether reinvesting depreciation increases the amount products of the production process. Later, the same problem was discussed by Ruchti and Lohmann in 1940s. Hence, this effect was named as the Marx-Engles effect or the Ruchti-Lohamann effect. Stendl(1952) paid attention to this effect in discussing the prosperity and decline of the post-war American capitalism. Therefore, we name this effect, including Kalecki(1954), as the Marx-Engles-Ruchti-Lohmann-Kalecki-Steindl effect, in short MERLKS, in what follows. We discuss the working mechanism of this effect with some simple examples.

4.2. Normal depreciation. In the simplest case with a constant fixed rate of depreciation, say the reciprocal of the durability of fixed capital, it is known that the total capacity of production lines will be increased, that this magnitude of increases in capacity is determined by durability alone, and that the top peak of increase comes in exactly the same with durability.

It is *assumed*, for the sake of simplicity, that fixed capital is divisible *ad infinitum*.

The following is a simple example of MERLKS with durability 5.

TABLE 2. MERLKS—Normal depreciation

period		1	2	3	4	5	6	7
capacity		1000	1200	1440	1728.0	2073.60	1488.320	1585.984
age	0	1000	200	240	288.0	345.60	414.720	297.664
	1		1000	200	240.0	288.00	345.600	414.720
	2			1000	200.0	240.00	288.000	345.600
	3				1000.0	200.00	240.000	288.000
	4					1000.00	200.000	240.000
depreciation		200	240	288	345.6	414.72	297.664	317.197

Reinvestment of depreciation of fixed capital will create fluctuations in the economy, and the cycle of this fluctuation will not be shorter than the twice of durability, although dynamics of this renewal process is converging to the stationary state. The stationary state of MERLKS is given by its multiplier $\beta = \frac{2m}{m+1}$, where m denotes durability. In the above example case, it is $\frac{10}{6} \times 1000 = 1666.66 \dots$

As simple calculation shows, the effect indicates that the amount of fixed capital will converge to the stationary value, when depreciation of fixed capital of each year is just invested.

The first apex comes in more or less the same year length with its durability. This means that the reinvestment of depreciation of fixed capital brings about the upward thrust or spike of the total capacity of fixed capital installed in the production process.

The length of periods in which the stationary state is reached is usually much longer than durability of fixed capital.

4.3. Yamada-Yamada's renewal dynamics. In Li-Fujimori(2013), some formal results of MERLKS are reviewed in view of Yamada-Yamada(1960) . Yamada-Yamada's system of equations is given by the next three:

$$(4.1) \quad D(t) = \frac{1}{m}K(t-1),$$

$$(4.2) \quad H(t) = F(t-m) + D(t-m),$$

$$(4.3) \quad K(t) = K(t-1) + F(t) + D(t) - H(t).$$

In what follows, we ignore net investment: $F(t) = 0$. Then, one obtains

$$K(t) - \left(1 + \frac{1}{m}\right)K(t-1) + \frac{1}{m}K(t-m-1) = 0.$$

Hence,

$$(4.4) \quad K(t+m+1) - \left(1 + \frac{1}{m}\right)K(t+m) + \frac{1}{m}K(t) = 0.$$

The characteristic equation of the above is expressed by

$$(4.5) \quad \lambda^{m+1} - \left(1 + \frac{1}{m}\right)\lambda^m + \frac{1}{m} = 0.$$

After factorisation, one obtains:

$$(\lambda - 1)^2 \left(\lambda^{m-1} + \frac{m-1}{m}\lambda^{m-2} + \dots + \frac{1}{m} \right) = 0.$$

Therefore, characteristic roots other than duplicated 1 are derived from the following:

$$(4.6) \quad \lambda^{m-1} + \frac{m-1}{m}\lambda^{m-2} + \dots + \frac{1}{m} = 0.$$

By applying Eneström-Kakeya's theorem on polynomial equations to (4.6), Yamada-Yamada proved that the dynamic path of fixed capital converges to a steady state, with diminishing oscillation.^{2, 3}

²As for Eneström-Kakeya's theorem, refer to *e.g.* Anderson-Saff-Varga(1979).

³The characteristic equation of the original Yamada-Yamada model has duplicated root 1. Hence, the Jordan form of the companion matrix of (4.5) includes a Jordan block $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; namely, the dynamics of the system is not free from the possibility of resonance.

Yamada-Yamada(1960) found that the characteristic equation of renewal dynamics of fixed capital has duplicated dominant roots, but did not refer to possibility of resonance caused by duplicate roots.⁴ Although renewal dynamics itself is converging, the problem is how soon it converges.

4.4. Accelerated depreciation. This subsection will see how the situation will be changed if accelerated depreciation takes place.

Most of analyses of fixed capital depend on the assumption that the production processes proceed normally as expected *ex ante*. That is, fixed capital will be used until it reaches its technical durability.

Let us *suppose* that durability of fixed capital is m years. Its depreciation, however, is done in the first $\tau = m - 1$ years. That is, fixed capital is operated without depreciation in the period of its final age. This is a simple case of accelerated depreciation.

Yamada-Yamada's equation will be replaced by the following:

$$(4.7) \quad H(t) = D(t - m),$$

$$(4.8) \quad K(t) = K(t - 1) + D(t) - H(t),$$

$$(4.9) \quad D(t) = \frac{1}{\tau} (K(t) - D(t - m - 1)).$$

From these, one gets

$$(4.10) \quad D(t) - \frac{\tau}{\tau - 1} D(t - 1) + \frac{1}{\tau - 1} D(t - m) + \frac{1}{\tau - 1} D(t - m - 1) - \frac{1}{\tau - 1} D(t - m - 2) = 0.$$

The characteristic equation of this is given by

$$(4.11) \quad \lambda^{m+2} - \frac{\tau}{\tau - 1} \lambda^{m+1} + \frac{1}{\tau - 1} \lambda^2 - \frac{1}{\tau - 1} \lambda - \frac{1}{\tau - 1} = 0.$$

This can be factorised as follows:

$$(4.12) \quad (\lambda - 1)^2 \left(\lambda^m + \frac{\tau - 2}{\tau - 1} \lambda^{m-1} + \frac{\tau - 3}{\tau - 1} \lambda^{m-2} + \dots + \frac{\tau - 3}{\tau - 1} \right) = 0.$$

The next example is a modification of the above Table 2 to show accelerated depreciation. We *assume* that fixed capital operates for 5 periods, but depreciation

⁴ The matrix for renewal dynamics of fixed capital is similar to that of population dynamics, known as the Leslie matrix.

will be made in 4 periods. The rate of depreciation is set to a constant, *i.e.* $\frac{1}{4}$. Hence, every equipment is operated in the last period of its life without keeping any economic value. Besides, we *ignore* the tax system.

TABLE 3. MERLKS—Accelerated depreciation

period	1	2	3	4	5	6	7
capacity	1000	1250	1562.5	1953.125	2441.406	1488.32	1939.697
age	0	1000	250	312.5	390.625	488.281	360.352
	1		1000	250.0	312.500	390.625	488.281
	2			1000.0	250.000	312.500	390.625
	3				1000.000	250.000	312.500
	4					1000.000	250.000
depreciation	250	312.5	390.625	488.281	360.352	387.939	406.799

Remark that fixed capital of age 4 is counted for as a part of capacity, but it has no value, and hence never enters into the object of depreciation any longer.

As is easily seen from tables, accelerated depreciation yields a larger increase in capacity for longer periods than otherwise. Accelerated depreciation reveals to be something like *positive externalities*.

It seems that all of firms know, however, that their equipments will sooner or later confront with new equipments with new technology. Hence, the accelerated amortisation method is often employed by firms, because most of firms are threatened by the advent of new technology.

Remark that the multiplier of MERLKS β depends on the period of depreciation, and not on durability. In the above case one has $\beta = \frac{2\tau}{2\tau + 1} \cdot \frac{\tau + 1}{\tau} = 2$.

5. Concluding Remarks

We dealt with depreciation of fixed capital from the angle of its influence on growth and fluctuation of the economy.

In 1960s, many Japanese Marxian economists discussed MERLKS in relation to the theory of crisis. Many of them were then based on value-ridden scheme of production by Marx, so that they seem to deny the influence of MERLKS to economic fluctuations in the end. They seem to regard depreciation only as cost.

They relied on the fact that the amount of fixed capital converges in the long-run. They did not notice that the true problem is how soon it converges.

If we take into account that the convergence time is greater than durability, MERLKS should not be ignored in considering the dynamic process, the part of which is a crisis.

In this section, we summarise the points which were taken up with respect to fixed capital in this short article.

As we saw in the above, depreciation has two faces. Is depreciation cost or profit? The answer might be 'both.' Fixed capital possesses two faces (Paton-Littleton, 1940); in one side, depreciation is a cost factor, and in the other, depreciation becomes a part of the source for investment.

Now, the relationship among the rate of profit, the rate of growth and the rate of accumulation is an important point in discussing economic theory. Our analysis shows that depreciation should be treated as a part of the source for investment, and we can reestablish the so-called Cambridge equation. The Cambridge equation is considered in the brandnew commodity world seems to give a good explanation of economies, say China's economy in the past 20 years. We will elaborate on this point in near future.

If we focus on reinvestment of depreciation, we immediately see that it will shift the stationary state of the economy upward, to the one with higher level of production capacity from the initial one, which is called the MERLKS effect. This movement is a converging oscillation. Its convergence is slow, accompanied by spikes of capacity. Hence, one should pay attention to this nature of fixed capital; fixed capital is an indispensable basis of production, but, at the same time, investment of fixed capital brings about disequilibrium in the economy.

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