# Complete Division of Labour and Generalized Theory of Value: A New Framework Based on Agents' Two-Stage Decisions

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ABSTRACT. This paper constructs a framework for the generalized theory of value by means of agents' two-stage decision approach, in which an agent at first makes his/her production decision to maximize total revenue, and consumption decision to maximize utility thereafter. We endogenize configuration and structure of division of labour into individual choice behaviour in our general equilibrium setting. We take into account not only market-clearing conditions, but also the allocation condition of labour force between production sectors of restrictive capacities, *i.e.* the principle of equivalent rate of comparative advantage. By virtue of production functions' return-to-scale properties and endogenous/exogenous comparative advantage thereof, different division-of-labour systems are defined, and therefore described by this same theory of value which concludes that, for any type of division of labour, Marx's labour theory of value is a special case of ours.

*Keywords*: Complete division of labour; generalized theory of value; agent's two-stage decisions

## 1. Introduction

Generalized Theory of Value (GTV) has been studied preliminarily for almost 30 years. It was constructed by Cai, J. in 1985, formed by Cai, J. and Li, R. in 2001, and developed by Cai, J. and Jiang, Y. in 2009 and 2010; and now, it has been a comparatively mature and extraordinary theory of value. However, GTV is a developing theory and should be demonstrated in further. Thanks to scholarsâĂŹ critics and queries about GTV,<sup>1</sup> which not only drive us to complete the theory, but also supply us reference for follow-up study. Hence, this paper supplements and demonstrates the original theory from the following aspects.

*Firstly*, it is considered that the Ricardian model of comparative advantage hypothesizes the international immobility of labour, which applies to international exchange of goods only, but not suits the case of mobility of labour between domestic sectors. According to it, GTV based on the Ricardian model of comparative

<sup>&</sup>lt;sup>1</sup> See Liu Yuxun(2005), and Yue Hongzhi, Kou Yaling(2005), etc.

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advantage must demonstrate that it applies to and even firstly applies to domestic exchange of goods. By means of agent's individual framework, this paper proves that comparative advantages exist between individuals, and that the Ricardian model of comparative advantage applies to variable types of the division-of-labour system.

Secondly, as the core proposition of GTV, the "principle of equivalent rate of comparative advantage," *PERCA*, is a controversial focus. Although Cai Jiming and Jiang Yongji (2009) demonstrated it with the existence of equilibrium and its stability, it just involved in variable division-of-labour system, and the way of argumentation was limited in the linear production function setting. This paper expands the principle of equivalent rate of comparative advantage to a non-linear production function setting and kinds of division-of-labour systems derived from it.

*Finally*, a logically rigorous theory of value must stand on its own rational basis. Therefore, this paper attempts to provide one with logical consistency for GTV to expect recognition from more economists in and abroad.

Therefore, this paper adopts agents' two-stage decision approach and starts with agent's production decision and consumption decision to construct the micro framework of GTV by means of general equilibrium conditions of markets. In addition, by virtue of production functions' return-to-scale properties and endogenous/exogenous comparative advantage thereof, different division-of-labour systems are defined.

### 2. General Equilibrium Framework of Generalized Theory of Value

**2.1. Description of model.** We assume a closed economy, in which the population of economic participants is  $\overline{M}$ , which means that everyone in this system is both a producer and consumer, in short an *agent* in this article, and they produce and consume two types of goods named x and y; each agent must consider two-stage decisions: the production decision maximizing the total revenue and the consumption decision maximizing utility. Agents can be divided into different types according to the production decision and consumption decision, and the population of each type is  $M_i$ , and the same type of agents acts the production and consumption behavior in the same way. For each agent of type *i*, called agent *i*, pairs of  $(x_i, y_i)$ ,  $(x_i^d, y_i^d)$  and  $(x_i^s, y_i^s)$  respectively stand for the quantity of self-sufficiency, quantity demanded and supplied of the two goods in markets. Hence, the quantities of production of the two goods are  $x_i^p = x_i + x_i^s$ ,  $y_i^p = y_i + y_i^s$ , respectively; the quantities of consumption of the two goods are  $x_i^c = x_i + x_i^d$ ,  $y_i^c = y_i + y_i^d$ , respectively; the value of good j ( $j \in \{x, y\}$ ) is denoted as  $v_j$ , thus agent *i*'s total value of production is  $Q_i = v_x x_i^p + v_y y_i^p$ ; the C-D function setting of its individual utility

 $u_i$  is determined by the quantities of consumption of the two goods and preference parameter  $\beta_i \in (0, 1)$ ;<sup>2</sup> for the production technology, agent *i*'s production function corresponding to good *j* is  $f_{ij}(l_{ij})$ , where  $f_{ij}(\cdot)$  is monotonically increasing and twice continuously differentiable, with  $f'_{ij} > 0$ ,  $f''_{ij} \ge 0$  and  $f_{ij}(0) = 0$ , where  $l_{ij}$ denotes the labour time spent in good *j*'s production, and labour endowment is 1; an individual budget line requests that the total value of the supplied goods equals the demanded goods. Hence, agent *i*'s production and consumption decisions are described as in what follows.

## **2.2.** Agent *i*'s production decision. It is to choose $\{l_{ix}, l_{iy}\}$ such that

$$\max Q_i = v_x x_i^p + v_y y_i^p$$
  
s.t.  $x_i^p = f_{ix}(l_{ix}),$   
 $y_i^p = f_{iy}(l_{iy}),$   
 $l_{ix} + l_{iy} = 1.$ 

By solving the decision problem above, one gets that:  $\frac{dQ_i}{dl_{ix}} = v_x f'_{ix}(l_{ix}) - v_y f'_{iy}(1 - l_{ix}); \frac{d^2Q_i}{dl_{ix}^2} = v_x f''_{ix}(l_{ix}) + v_y f''_{iy}(1 - l_{ix}) \ge 0$ . Let  $v \equiv \frac{v_x}{v_y}$ .  $\frac{dQ_i}{dl_{ix}} > 0$ , if and only if  $v > \frac{f'_{iy}(l_{iy})}{f'_{ix}(l_{ix})}$ , which means that the total value of production increases accompanied  $d \frac{f'_{iy}(l_{iy})}{f'_{ix}(l_{ix})}$ 

with an increase in  $l_{ix}$ . Since  $\frac{d \frac{f'_{iy}(l_{iy})}{f'_{ix}(l_{ix})}}{dl_{ix}} < 0$ , in order to maximize total value of production, agent *i* eventually chooses the situation  $l_{ix} = 1$ , that is, agent *i* will specialize in producing good *x*.

Similarly, if and only if  $v < \frac{f'_{iy}(l_{iy})}{f'_{ix}(l_{ix})}$ , agent *i* eventually chooses the situation  $l_{ix} = 0$ , which means that agent *i* will specialize in producing good *y*.

According to the above, we obtain the following proposition:

PROPOSITION 2.1. Let  $f_{ij}(\cdot)$  stand for a function of  $l_{ij}$  with degree  $\gamma_{ij} \ge 1$ . If  $\frac{\gamma_{iy}f_{iy}(l_{iy})l_{iy}^{-1}}{\gamma_{ix}f_{ix}(l_{ix})l_{ix}^{-1}} < \frac{v_x}{v_y} < \frac{\gamma_{i'y}f_{i'y}(l_{i'y})l_{i'y}^{-1}}{\gamma_{i'x}f_{i'x}(l_{i'x})l_{i'x}^{-1}}$ , agent *i* (resp. *i'*) specializes in producing good *x* (resp. *y*), where  $i \ne i'$ . If  $\frac{\gamma_{iy}f_{iy}(l_{iy})l_{iy}^{-1}}{\gamma_{ix}f_{ix}(l_{ix})l_{ix}^{-1}} > \frac{v_x}{v_y} > \frac{\gamma_{i'y}f_{i'y}(l_{i'y})l_{i'y}^{-1}}{\gamma_{i'x}f_{i'x}(l_{i'x})l_{ix}^{-1}}$ , agent *i* (resp. *i'*) specializes in producing good *y* (resp. *x*).

<sup>&</sup>lt;sup>2</sup> It is necessary to remark that this paper brings in utility functions to endogenize the individual supply and demand but not adopts the so called utility theory of value; in the model, value is decided by labour time, which is demonstrated by "aggregate social product equates total social labour time" in the section of "general equilibrium system of market economy."

 $\begin{array}{l} \textit{Proof.} (1) \text{ Since } f_{ij}(\cdot) \text{ is a function of } l_{ij} \text{ with degree } \gamma_{ij} \geq 1, \text{ one has by differenti-}\\ \text{ation } f'_{ij}(l_{ij}) l_{ij} = \gamma_{ij} f_{ij}(l_{ij}) \text{ i.e. } f'_{ij}(l_{ij}) = \frac{\gamma_{ij} f_{ij}(l_{ij})}{l_{ij}}, \text{ then } \frac{f'_{iy}(l_{iy})}{f'_{ix}(l_{ix})} = \frac{\gamma_{iy} f_{iy}(l_{iy})l_{iy}^{-1}}{\gamma_{ix} f_{ix}(l_{ix})l_{ix}^{-1}};\\ \text{hence, if } v > \frac{\gamma_{iy} f_{iy}(l_{iy})l_{iy}^{-1}}{\gamma_{ix} f_{ix}(l_{ix})l_{ix}^{-1}}, \text{ then } l_{ix} = 1; \text{ if } v < \frac{\gamma_{iy} f_{iy}(l_{iy})l_{iy}^{-1}}{\gamma_{ix} f_{ix}(l_{ix})l_{ix}^{-1}}, \text{ then } l_{ix} = 0;\\ \text{(2) Similarly, for agent } i', \text{ if } v > \frac{\gamma_{i'y} f_{i'y}(l_{i'y})l_{i'y}^{-1}}{\gamma_{i'x} f_{i'x}(l_{i'x})l_{i'x}^{-1}}, \text{ then } l_{i'x} = 1; \text{ on the contrary,}\\ l_{i'x} = 0 \text{ follows, if } v < \frac{\gamma_{i'y} f_{i'y}(l_{i'y})l_{i'y}^{-1}}{\gamma_{i'x} f_{i'x}(l_{i'x})l_{i'x}^{-1}}, \end{array}$ 

(3) According to the above (1) and (2), if  $\frac{\gamma_{iy}f_{iy}(l_{iy})l_{iy}^{-1}}{\gamma_{ix}f_{ix}(l_{ix})l_{ix}^{-1}} < v < \frac{\gamma_{i'y}f_{i'y}(l_{i'y})l_{i'y}^{-1}}{\gamma_{i'x}f_{i'x}(l_{i'x})l_{ix}^{-1}}$ , then agent *i* specializes in producing good *x*, and agent *i'* specializes in producing good *y*.

Analogously, if  $\frac{\gamma_{iy} f_{iy}(l_{iy}) l_{iy}^{-1}}{\gamma_{ix} f_{ix}(l_{ix}) l_{ix}^{-1}} > v > \frac{\gamma_{i'y} f_{i'y}(l_{i'y}) l_{i'y}^{-1}}{\gamma_{i'x} f_{i'x}(l_{i'x}) l_{i'x}^{-1}}$ , agent *i* specializes in producing good *y*, and agent *i'* specializes in producing good *x*.

**2.3. Agent i's consumption decision.** One has to choose variables  $\{x_i, y_i, x_i^s, y_i^s, x_i^d, y_i^d\}$  to make:  $u_i = (x_i^c)^{\beta_i} (y_i^c)^{1-\beta_i}, x_i^c = x_i + x_i^d, y_i^c = y_i + y_i^d, x_i^p = x_i + x_i^s, y_i^p = y_i + y_i^s$  and  $vx_i^s + y_i^s = vx_i^d + y_i^d$ .

Proposition 2.1 describes a sufficient condition of division of labour for agent *i* and *i'*. However, for the convenience of deduction, we call it agent 1 which specializes in producing good *x* and agent 2 which specializes in producing good *y*. Therefore, agent 1 specializes in producing good *x* and its quantity satisfies  $x_1^p = f_{1x}(1)$ , it also supplies market the rest  $x_1^s$  after self-consumption to buy demanded  $y_1^d$ ; agent 2 specializes in producing good *y* and its quantity satisfies  $y_2^p = f_{2y}(1)$ , it also supplies market the rest  $y_2^s$  after self-consumption to buy demanded  $x_2^d$ . Therefore, the agents' consumption decision is rewritten as follows:

Agent 1's consumption decision:

$$\max_{\{x_1, x_1^s, y_1^d\}} u_1 = x_1^{\beta_1} (y_1^d)^{1-\beta_1}$$

s.t.

$$x_1 + x_1^s = f_{1x}(1),$$
  
 $vx_1^s = y_1^d.$ 

It is concluded that:

$$\begin{cases} x_1 = \beta_1 f_{1x}(1), \\ x_1^s = (1 - \beta_1) f_{1x}(1) , \\ y_1^d = v(1 - \beta_1) f_{1x}(1). \end{cases}$$

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Agent 2's consumption decision:

$$\max_{\{x_2^d, y_2, y_2^s\}} u_2 = (x_2^d)^{\beta_2} y_2^{1-\beta_2}$$

s.t.

$$y_2 + y_2^s = f_{2y}(1),$$
  
 $y_2^s = v x_2^d.$ 

It is concluded that:

$$\begin{cases} x_2^d = \frac{1}{v}\beta_2 f_{2y}(1), \\ y_2 = (1 - \beta_2) f_{2y}(1), \\ y_2^s = \beta_2 f_{2y}(1). \end{cases}$$

**2.4. General equilibrium system of market economy.** General equilibrium system of market economy must satisfy the conditions as follows: *Firstly*, the market is cleared, which means total good x supplied by agent 1 must equate total good x demanded by agent 2, as well as that total demanded good y by agent 1 must equate total supplied good y by agent 2. *Secondly*, total value must equate total labour time, which means aggregate social products must equate total social labour time in the model of  $2 \times 2 \times 1$  (two-sector, two-good and one-labour). *Thirdly*, allocation of labour between two sectors follows the *PERCA*, which connotes that limited capacity of production sector limits demands of labour to prevent agents from accessing market freely to maximize own utility, but to compete labour by maximized quantity of comparative advantage. *Finally*, population constraints: the total quantity of two kinds of agent, which means the total quantity of labour of two sectors must equate the total population in the economic system.

#### **Conditions of clearing markets.**

$$\begin{cases} M_1 x_1^s = M_2 x_2^d, \\ M_1 y_1^d = M_2 y_2^s. \end{cases}$$

It follows from the conditions of clearing markets above:

(2.1) 
$$v \equiv \frac{v_x}{v_y} = \frac{\beta_2}{1 - \beta_1} \frac{f_{2y}(1)}{f_{1x}(1)} \frac{M_2}{M_1}$$

Aggregate social production equates total social labour time.

(2.2) 
$$M_1 v_x x_1^p + M_2 v_y y_2^p = M_1 + M_2$$

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It is concluded from (2.1) and (2.2):

(2.3) 
$$\begin{cases} v_x = \frac{\beta_2}{1-\beta_1+\beta_2} \frac{1}{f_{1x}(1)} \left(1 + \frac{M_2}{M_1}\right), \\ v_y = \frac{1-\beta_1}{1-\beta_1+\beta_2} \frac{1}{f_{2y}(1)} \left(1 + \frac{M_1}{M_2}\right). \end{cases}$$

It is concluded by (2.3):

$$\begin{pmatrix} v_x - \frac{1}{f_{1x}(1)} = \frac{\beta_2}{1 - \beta_1 + \beta_2} \frac{1}{f_{1x}(1)} \left( \frac{M_2}{M_1} - \frac{1 - \beta_1}{\beta_2} \right), \\ v_y - \frac{1}{f_{2y}(1)} = \frac{1 - \beta_1}{1 - \beta_1 + \beta_2} \frac{1}{f_{2y}(1)} \left( \frac{M_1}{M_2} - \frac{\beta_2}{1 - \beta_1} \right). \end{cases}$$

We have the following:

PROPOSITION 2.2.  $v_x = \frac{1}{f_{1x}(1)}$  and  $v_y = \frac{1}{f_{2y}(1)}$ , iff  $\frac{M_2}{M_1} = \frac{1-\beta_1}{\beta_2}$ .

Proposition 2.2 indicates that only if  $\frac{M_2}{M_1} = \frac{1-\beta_1}{\beta_2}$ , the value of individual good equates average labour expenditure of individual good, which means that the labour theory of value holds.

## Principle of equivalent rate of comparative advantage.<sup>3</sup>

That the rate of comparative advantage of the two types of agents equates can be expressed as:

$$\frac{y_1^d - y_1^{oc}}{y_1^d} = \frac{x_2^d - x_2^{oc}}{x_2^d}.$$

Multiply  $M_1$  to both the numerator and the denominator in the left-hand side and  $M_2$  to both the numerator and the denominator in the right-hand side. Then, the condition of clearing markets is substituted, and the formula above is rewritten as:

$$\frac{M_2 y_2^s - M_1 y_1^{oc}}{M_2 y_2^s} = \frac{M_1 x_1^s - M_2 x_2^{oc}}{M_1 x_1^s}$$

where,  $y_1^{oc}$  and  $x_2^{oc}$  respectively stand for agent 1's opportunity cost producing good y itself and agent 2's opportunity cost producing good x itself. For agent 1, the opportunity cost of labour to produce good y is the labour spent to sale good x applied to buy good y, thus the opportunity cost of labour to produce good y is the quantity of the good y the labour can produce. Let  $l_{1y}^{oc}$  stand for the opportunity cost of labour to produce good x applied to buy good y, and there is  $l_{1y}^{oc} \equiv l_{1x}^s$ . Since  $l_{1x}^s = f_{1x}^{-1}(x_1^s)$ , opportunity cost to produce good y  $y_1^{oc}$  satisfies  $y_1^{oc} \equiv f_{1y}(f_{1x}^{-1}(x_1^s))$ . Similarly, for agent 2, the opportunity cost of labour to produce good x is the labour spent to sell good y

 $<sup>^{3}</sup>$  In fact, it is rigorously demonstrated that *PERCA* works, whether the production function is linear or nonlinear; in addition, the rate of comparative advantage and resulting allocation of labour are brought about by competitive equilibrium, and the Marshallian condition of stability is met. However, proof is omitted.

applied to buy good x, thus the opportunity cost of labour to produce good x is the quantity of the good x the labour can produce; and opportunity cost to produce good x  $x_2^{oc}$  satisfies  $x_2^{oc} \equiv f_{2x}(f_{2y}^{-1}(y_2^s))$ . Substituting  $y_1^{oc} \equiv f_{1y}(f_{1x}^{-1}(x_1^s))$  and  $x_2^{oc} \equiv f_{2x}(f_{2y}^{-1}(y_2^s))$  into the formula above, we have that:

(2.4) 
$$\frac{M_2}{M_1} = \left[\frac{(1-\beta_1)f_{1x}(1)f_{1y}(f_{1x}^{-1}(1-\beta_1)f_{1x}(1))}{\beta_2 f_{2y}(1)f_{2x}(f_{2y}^{-1}(\beta_2 f_{2y}(1)))}\right]^{\frac{1}{2}}.$$

**Population constraints.** 

(2.5) 
$$M_1 + M_2 = M$$
.

From (2.4) and (2.5), one obtains the following:

$$\begin{cases} M_{1} = \frac{1}{1 + \left[\frac{(1-\beta_{1})f_{1x}(1)f_{1y}(f_{1x}^{-1}(1-\beta_{1})f_{1x}(1))}{\beta_{2}f_{2y}(1)f_{2x}(f_{2y}^{-1}(\beta_{2}f_{2y}(1))}\right]^{\frac{1}{2}}\bar{M}, \\ M_{2} = \frac{\left[\frac{(1-\beta_{1})f_{1x}(1)f_{1y}(f_{1x}^{-1}(1-\beta_{1})f_{1x}(1))}{\beta_{2}f_{2y}(1)f_{2x}(f_{2y}^{-1}(\beta_{2}f_{2y}(1))}\right]^{\frac{1}{2}}}{1 + \left[\frac{(1-\beta_{1})f_{1x}(1)f_{1y}(f_{1x}^{-1}(1-\beta_{1})f_{1x}(1))}{\beta_{2}f_{2y}(1)f_{2x}(f_{2y}^{-1}(\beta_{2}f_{2y}(1))}\right]^{\frac{1}{2}}\bar{M}. \end{cases}$$

Substituting (2.4) into (2.3), then it is concluded that:

$$\begin{cases} v_x = \frac{\beta_2}{1-\beta_1+\beta_2} \frac{1}{f_{1x}(1)} \{1 + \left[\frac{(1-\beta_1)f_{1x}(1)f_{1y}(f_{1x}^{-1}(1-\beta_1)f_{1x}(1))}{\beta_2 f_{2y}(1)f_{2x}(f_{2y}^{-1}(\beta_2 f_{2y}(1))}\right]^{\frac{1}{2}}\},\\ v_y = \frac{1-\beta_1}{1-\beta_1+\beta_2} \frac{1}{f_{2y}(1)} \{1 + \left[\frac{(1-\beta_1)f_{1x}(1)f_{1y}(f_{1x}^{-1}(1-\beta_1)f_{1x}(1))}{\beta_2 f_{2y}(1)f_{2x}(f_{2y}^{-1}(\beta_2 f_{2y}(1))}\right]^{-\frac{1}{2}}\}.\end{cases}$$

The functions above are general production functions which gives the complete framework of GTV without an individual setting about production functions.

## 3. Return-to-Scale and Comparative Advantage

According to Proposition 2.1, sufficient conditions of agent 1 specializing in producing good x and agent 2 specializing in producing good y are:  $\frac{\gamma_{iy} f_{iy}(l_{iy}) l_{iy}^{-1}}{\gamma_{ix} f_{ix}(l_{ix}) l_{i'x}^{-1}} < \frac{v_x}{v_y} < \frac{\gamma_{i'y} f_{i'y}(l_{i'y}) l_{i'y}^{-1}}{\gamma_{i'x} f_{i'x}(l_{i'x}) l_{i'x}^{-1}} \Rightarrow \frac{f_{ix}(l_{ix}) l_{ix}^{-1}}{f_{iy}(l_{iy}) l_{iy}^{-1}} > \frac{\gamma_{iy} \gamma_{i'x}}{\gamma_{ix} \gamma_{i'y}};$  sufficient conditions of agent *i* specializing in producing good *y* and agent *i'* specializing in producing good *x* and agent *i* specializing in producing good *y* and agent *i'* specializing in producing good *x* and agent *i'* specializing in producing good *y* and agent *i'* specializing in producing good *x* and agent *i'* specializing in producing good *y* and agent *i'* specializing in producing good *y'* an

$$RP_{i,i'} \equiv \frac{f_{ix}(l_{ix})l_{ix}^{-1}}{f_{iy}(l_{iy})l_{iy}^{-1}} \cdot \frac{f_{i'y}(l_{i'y})l_{i'y}^{-1}}{f_{i'x}(l_{i'x})l_{i'x}^{-1}}.$$

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If  $\Gamma \equiv \frac{\gamma_{iy}\gamma_{i'x}}{\gamma_{ix}\gamma_{i'y}} > 0$ , then two conditions above can be rewritten as:  $RP_{i,i'} \leq \Gamma$ , which means if  $RP_{i,i'} > \Gamma$ , then agent *i* specializes in producing good *x* and agent *i'* specializes in producing good *y*; if  $RP_{i,i'} < \Gamma$ , then agent *i* specializes in producing good *x*; however, if  $RP_{i,i'} = \Gamma$ , agents are indifferent as to specialisation.

If  $f_{ij}(\cdot)$  is linear in  $l_{ij}$ , *i.e.* constant returns to scale, then labour productivity satisfies  $\frac{f_{ij}(l_{ij})}{l_{ij}} = f_{ij}(1)$ , and it is constant for a given production function, thus  $RP_{i,i'} = \frac{f_{ix}(1)}{f_{iy}(1)} \cdot \frac{f_{i'y}(1)}{f_{i'x}(1)}$ . Especially on condition of the Ricardian model:  $f_{ij}(l_{ij}) = \alpha_{ij}l_{ij}$ , where a production coefficient  $\alpha_{ij} > 0$ , and  $RP_{i,i'} = \frac{\alpha_{ix}\alpha_{i'y}}{\alpha_{i'x}\alpha_{iy}}$ .

According to  $\frac{\partial RP_{i,i'}}{\partial l_{ij}} = 0$ , the discriminant of relative productivity will not change together with changes of labour input, which means that the variety of labour input does not influence comparative advantage between agents, or directions of complete division of labour thereafter. What influences comparative advantage and directions of division of labour is relative productivity between two agents, on condition that it is an exogenous production function, or, taking Ricardian model for granted, an exogenous production coefficient. It means that comparative advantages and directions of complete division of labour between agents are determined initially. Yang, X. and J. Borland (1991) called the comparative advantage "basing on external given differences on technology and endowment between individuals" the "exogenous comparative advantage." (Yang, X. and Huang, Y, 1999, p.6)

If  $f_{ij}(\cdot)$  is a function of  $l_{ij}$  with degree  $\gamma_{ij} > 1$ , *i.e.* increasing returns to scale,  $f'_{ij}(l_{ij})l_{ij} = \gamma_{ij}f_{ij}(l_{ij}) > f_{ij}(l_{ij}) \Rightarrow f'_{ij}(l_{ij}) > f_{ij}(l_{ij})l_{ij}^{-1}$ , then:  $\frac{df_{ij}(l_{ij})l_{ij}^{-1}}{dl_{ij}} = l_{ij}^{-1} \left[ f'_{ij}(l_{ij}) - \frac{f_{ij}(l_{ij})}{l_{ij}} \right] > 0$ ;  $\frac{df_{ij'}(l_{ij'})(l_{ij'})^{-1}}{dl_{ij}} = -l_{ij'}^{-1} \left[ f'_{ij'}(l_{ij'}) - \frac{f_{ij'}(l_{ij'})}{l_{ij'}} \right] < 0$ , which means that with the increase of labour input to good j, agent i's labour productivity about good j rises in contrast to that its labour productivity about the other good decreases.<sup>4</sup> Therefore, agent i's comparative advantage about good j

$$\frac{d\left(\frac{f_{ij}(l_{ij})l_{ij}^{-1}}{f_{ij'}(l_{ij'})l_{ij'}^{-1}}\right)}{dl_{ij}} > 0 \Leftrightarrow \frac{d\left(\frac{f_{ij'}(l_{ij'})l_{ij'}^{-1}}{f_{ij}(l_{ij})l_{ij}^{-1}}\right)}{dl_{ij}} < 0.$$

Let j = x and j' = y. On condition of constant labour input of agent i',  $\frac{\partial RP_{i,i'}}{\partial l_{ix}} > 0 \Leftrightarrow \frac{\partial RP_{i,i'}}{\partial l_{iy}} < 0$ , which means that the discriminant of relative productivity changes accompanied with the variety of labour input influencing agents on comparative advantage and directions of complete division of labour. In addition, the above studies proves the following: if  $RP_{i,i'} > \Gamma$ , then agent *i* has comparative

<sup>&</sup>lt;sup>4</sup> Yang, X. (2003, p.36) called this phenomena as "economics of specialization."

advantage to produce good x and agent i' has comparative advantage to produce good y, thus agent i inclines to input more  $l_{ix}$  and agent i' inclines to input more  $l_{i'y}$ ; in such a case,  $RP_{i,i'}$  increases continuously and the situation of  $RP_{i,i'} > \Gamma$ holds; if  $RP_{i,i'} < \Gamma$ , then agent i has comparative advantage to produce good y and agent i' has comparative advantage to produce good x, thus agent i inclines to input more  $l_{iy}$  and agent i' inclines to input more  $l_{i'x}$ ; in case of it,  $RP_{i,i'}$  decreases continuously and the situation  $RP_{i,i'} < \Gamma$  remains. Therefore, reversal of direction of comparative advantage will not happen, neither dose the direction of complete division of labour, thereafter.

Yang and Huang(1999, p.5) considered that "on condition that the same individual labour in prior chooses to produce different products in different specialization level, if increasing returns to specialization exits, comparative advantage may exist." The comparative advantage "basing on productivity difference of individual decision about specialization level" (Yang, X. and Huang, Y., 1999, p.31) is called "endogenous comparative advantage." Further, according to increasing returns to scale, agent *i* inputs all labour to produce good *j* and its productivity about good *j'* is 0; agent *i'* inputs all labour to produce good *j'* and its productivity about good *j* is 0; on condition of that, there is a complete division of labour and endogenous comparative advantage to produce good *x* and agent *i'* has absolute advantage to produce good *y*.

The above can be summarised as follows:

PROPOSITION 3.1. If a production function is linear in labour input, then changes of labour productivity will cause reversal of comparative advantage; if a production function is the one with degree  $\gamma_{ij}$  (> 1), then variety of labour productivity will not cause reversal of comparative advantage.

#### 4. Division of Labour System and Generalized Theory of Value

**4.1. Definitions of systems of division of labour.** According to production functions' return-to-scale properties and their corresponding relationships to endogenous/exogenous comparative advantage, as well as Proposition 3.1, we first define the variable division-of-labour system, the fixed division-of-labour system and the mixed division-of-labour system, and construct GTV in virtue of these different division-of-labour systems.

A structure of division of labour, in which the variety of agents' labour productivity causes reversal of comparative advantage, influences the direction of specialization on the structure of division of labour thereafter, is called the *variable*  *division-of-labour system*. A structure of division of labour, in which the variety of agents' labour productivity does not cause reversal of comparative advantage and the direction of specialization will not change thereafter, is called the *fixed division-of-labour system*. A structure of division of labour with properties of the above two systems, is called the *mixed division-of-labour system*. (Yang, 2001, pp.152-5; Yang, 2003, pp.119-22).

**4.2. Variable division-of-labour system.** According to Proposition 3.1 and above definitions, in a variable division-of-labour system, agent *i*'s production function corresponding to good *j* satisfies  $f_{ij}(l_{ij}) = \alpha_{ij}l_{ij}$ , i = 1, 2, j = x, y; in addition, since it is hypothesized that agent 1 specializes in producing good *x* and agent 2 specializes in producing good *y*, let us assume  $\frac{\alpha_{1x}}{\alpha_{1y}} > \frac{\alpha_{2x}}{\alpha_{2y}}$ , which means  $RP_{1,2} = \frac{\alpha_{1x}\alpha_{2y}}{\alpha_{1y}\alpha_{2x}} > 1 = \Gamma$ .

Substituting this production function into general equilibrium conditions of individual agent's consumption decision and the market economy, we can get several important individual decision variables and general equilibrium variables, which are displayed in Table 1 and Table 2. Therein, (2.4) is rewritten as:

(4.1) 
$$\frac{M_2}{M_1} = \frac{1 - \beta_1}{\beta_2} \left(\frac{\alpha_{1x}\alpha_{1y}}{\alpha_{2x}\alpha_{2y}}\right)^{\frac{1}{2}}$$

Let us put  $CP_{1,2} \equiv \left(\frac{\alpha_{1x}\alpha_{1y}}{\alpha_{2x}\alpha_{2y}}\right)^{\frac{1}{2}} = \frac{M_2}{M_1}\frac{\beta_2}{1-\beta_1}$ , where  $CP_{1,2}$  stands for the "discriminant of relative productivity" (Cai Jiming, 2010, pp.71-8), and (4.1) is rewritten as:

$$\frac{M_2}{M_1} = \frac{1 - \beta_1}{\beta_2} C P_{1,2}.$$

According to Proposition 2.1, if  $CP_{1,2} \neq 1$ ,  $\frac{M_2}{M_1} \neq \frac{1-\beta_1}{\beta_2}$ , then the value of individual good does not equate to average labour expenditure of individual good, which means the labour theory of value does not hold. In addition, if  $1 - \beta_1 = \beta_2 \Rightarrow \beta_1 + \beta_2 = 1$ , which means two agents' preference about the good produced by themselves is the same, then generalised values are expressed as:

$$\begin{cases} v_x = \frac{1}{2}\alpha_{1x}^{-1}(1 + CP_{1,2}), \\ v_y = \frac{1}{2}\alpha_{2y}^{-1}(1 + CP_{1,2}^{-1}). \end{cases}$$

These formula indicate that generalised values of good x and good y,  $v_x$  and  $v_y$ , are determined by their own "average absolute cost and relative productivity level  $CP_{1,2}$ " (Cai, J., 2010, p.76). The conclusions above are consistent with the results basing on the original framework.

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**4.3. Fixed division-of-labour system.** According to the view of endogenous comparative advantage from Yang, X. and J. Borland, it is hypothesized that there is no difference between the two types of agents before input, which indicates that the two agents have the same production function of good x and good y, and their preference of the two goods is the same, too.

According to Proposition 3.1 and three definitions, in a fixed division-of-labour system, let agent *i*'s production function corresponding to good *j* be  $f_{ij}(l_{ij}) = l_{ij}^{\alpha_j}$ ,  $\alpha_j > 1$ , i = 1, 2, j = x, y; a necessary and sufficient condition that agent 1 specializing in producing good *x* (comparing to agent 2 specializing in producing good *y*) has comparative advantage to producing good *x* but comparative disadvantage to producing good *y* is  $l_{1x} > l_{2x} \iff l_{1y} < l_{2y}$ ). One obtains the next:

PROPOSITION 4.1. 
$$\frac{l_{1x}^{\alpha_x-1}}{l_{1y}^{\alpha_y-1}} > \frac{l_{2x}^{\alpha_x-1}}{l_{2y}^{\alpha_y-1}}$$
 for  $\alpha_x, \alpha_y > 1$ , iff  $l_{1x} > l_{2x}$  ( $\iff l_{1y} < l_{2y}$ ).

*Proof.* (1) Necessity: Assume  $\frac{l_{1x}}{l_{2x}} \leq 1$ , and  $\frac{l_{1y}}{l_{2y}} = \frac{1-l_{1x}}{1-l_{2x}} \geq 1$ , so that  $\left(\frac{l_{1y}}{l_{2y}}\right)^{\frac{\alpha_y-1}{\alpha_x-1}} \geq 1$ , because  $\frac{\alpha_y-1}{\alpha_x-1} > 0$ . Then,  $\frac{l_{1x}^{\alpha_x-1}}{l_{1y}^{\alpha_y-1}} > \frac{l_{2x}}{l_{2y}^{\alpha_y-1}} \Rightarrow \frac{l_{1x}}{l_{2x}} > \left(\frac{l_{1y}}{l_{2y}}\right)^{\frac{\alpha_y-1}{\alpha_x-1}} \geq 1$ , which contradicts the hypothesis. Thus,  $\frac{l_{1x}}{l_{2x}} > 1$ .

(2) Sufficiency: Since  $l_{1x} > l_{2x}$  and  $l_{1y} < l_{2y}$ , one has  $\frac{l_{1x}}{l_{2x}} > 1 > \frac{l_{1y}}{l_{2y}}$ . Take the logarithm, and  $\ln \frac{l_{1x}}{l_{2x}} > 0 > \ln \frac{l_{1y}}{l_{2y}} \Longrightarrow (\alpha_x - 1) \ln \frac{l_{1x}}{l_{2x}} > 0 > (\alpha_y - 1) \ln \frac{l_{1y}}{l_{2y}}$ . It follows that  $\frac{l_{1x}^{\alpha_x - 1}}{l_{2x}^{\alpha_x - 1}} > \frac{l_{1y}^{\alpha_y - 1}}{l_{2y}^{\alpha_y - 1}}$ , hence  $\frac{l_{1x}^{\alpha_x - 1}}{l_{2y}^{\alpha_y - 1}} > \frac{l_{2x}^{\alpha_x - 1}}{l_{2y}^{\alpha_y - 1}}$ .

Proposition 4.1 shows that, if two agents which are the same in a prior "choose different specialization level when producing one good, difference exists" (Yang, X. and Huang, Y., 1999, p.31.), and "[t]he difference on productivity caused by different specialization model stands for endogenous comparative advantage." (Yang, X., 2003, p.42.)

In addition, another endogenous-comparative-advantage model, called the fixed division of labour II, is considered: if agent *i*'s production function for good x satisfies  $f_{ix}(l_{ix}) = l_{ix}^{\alpha_x}, \alpha_x > 1$ , then its counterpart for good y satisfies  $f_{iy}(l_{iy}) = \alpha_y l_{iy}, \alpha_y > 0$ . This makes it easy to prove Proposition 4.1:  $\frac{l_{1x}^{\alpha_x}}{\alpha_y} > \frac{l_{2x}^{\alpha_x}}{\alpha_y} \iff l_{1x} > l_{2x}$ . Individual decision variables and general equilibrium variables obtained from the two models of production functions are respectively shown in the column of fixed division of labour I and II in Table 1 and 2.

It will be shown that even if an extreme hypothesis, such as  $1-\beta_1 = \beta_2$ , namely two types of agents have the same preference on two goods, is adopted, generalised values equate traditional labour values by chance. In fact, if  $1 - \beta_1 = \beta_2$ , then Proposition 2.2 is rewritten as: if and only if  $\frac{M_2}{M_1} = 1$ ,  $v_x = \frac{1}{f_{1x}(1)}$ , as well as  $v_y = \frac{1}{f_{2y}(1)}$ . On condition of the fixed division of labour I,  $\frac{M_2}{M_1} = \left[\frac{(1-\beta_1)\frac{\alpha_x+\alpha_y}{\alpha_x}}{\beta_2^{\frac{\alpha_x+\alpha_y}{\alpha_y}}}\right]^{\frac{1}{2}}$ , only if  $\alpha_x = \alpha_y$ , then  $\frac{M_2}{M_1} = 1$ ; on condition of the fixed division of labour II,  $\frac{M_2}{M_1} = \left[\frac{(1-\beta_1)\frac{1+\alpha_x}{\alpha_x}}{\beta_2^{1+\alpha_x}}\right]^{\frac{1}{2}}$ , only if  $\alpha_x = 1$ , then  $\frac{M_2}{M_1} = 1$ . On condition of the fixed division of labour II,  $\frac{M_2}{M_1} = \left[\frac{(1-\beta_1)\frac{1+\alpha_x}{\alpha_x}}{\beta_2^{1+\alpha_x}}\right]^{\frac{1}{2}}$ , only if  $\alpha_x = 1$ , then  $\frac{M_2}{M_1} = 1$ . On condition of the fixed division of labour I and II,  $\alpha_x = \alpha_y$  or  $\alpha_x = 1$  indicates that two agents have the same output elasticity with respect to two goods,

which means two goods are absolutely the same in production level. It is another extreme hypothesis. Therefore, on condition of the fixed division-of-labour system, the labour theory of value is still a special case of GTV.

**4.4. Mixed division-of-labour system.** Based on the production function in Sachs, Yang and Zhang (1999a, b), the mixed division-of-labour model is constructed.

Let agent *i*'s production functions corresponding to good *x* and good *y* satisfy  $f_{ix}(l_{ix}) = l_{ix}^{\alpha_{ix}}$  and  $f_{iy}(l_{iy}) = \alpha_{iy}l_{iy}$ ; agent *i*'s production functions corresponding to good *x* and good *y* satisfy  $f_{i'x}(l_{i'x}) = \alpha_{i'x}l_{i'x}$  and  $f_{i'y}(l_{i'y}) = l_{i'y}^{\alpha_{i'y}}$ . Here assume  $\alpha_{iy}, \alpha_{i'x} > 0$ ,  $\alpha_{ix}, \alpha_{i'y} > 1$ , and  $i \neq i'$ . Therefore, the comparative-advantage relationship of them satisfies:  $\frac{l_{ix}^{\alpha_{ix}-1}}{\alpha_{iy}} \leq \frac{\alpha_{i'x}}{l_{i'y}^{\alpha_{i'y}-1}} \Longrightarrow l_{ix}^{\alpha_{ix}-1} l_{i'y}^{\alpha_{i'y}-1} \leq \alpha_{i'x}\alpha_{iy}$ .

According to Proposition 2.1, if  $l_{ix}^{\alpha_{ix}-1} l_{i'y}^{\alpha_{i'y}-1} > \alpha_{i'x} \alpha_{iy}$ , then agent *i* (resp. *i'*) specializes in producing good *x* (resp. *y*); if  $l_{ix}^{\alpha_{ix}-1} l_{i'y}^{\alpha_{i'y}-1} < \alpha_{i'x} \alpha_{iy}$ , then agent *i* (resp. *i'*) specializes in producing good *y* (resp. *x*). Therefore, if  $\alpha_{i'x} \alpha_{iy} < 1$  and  $l_{ix}^{\alpha_{ix}-1} l_{i'y}^{\alpha_{i'y}-1} > \alpha_{i'x} \alpha_{iy}$ , then the division-of-labour structure called mixed division of labour I is generated; if  $\alpha_{i'x} \alpha_{iy} > 1$  and  $l_{ix}^{\alpha_{ix}-1} l_{i'y}^{\alpha_{i'y}-1} \leq 1 < \alpha_{i'x} \alpha_{iy}$ , or  $\alpha_{i'x} \alpha_{iy} < 1$  but  $l_{ix}^{\alpha_{ix}-1} l_{i'y}^{\alpha_{i'y}-1} < \alpha_{i'x} \alpha_{iy}$ , then agent *i*''s division-of-labour structure called the mixed division of labour II is generated.

On condition of the mixed division of labour I, the production function of agent 1 specializing in producing good x satisifies  $f_{1x}(l_{1x}) = l_{1x}^{\alpha_{1x}}$ , and the production function of agent 2 specializing in producing good y satisifies  $f_{2y}(l_{2y}) = l_{2y}^{\alpha_{2y}}$ , which is the same as the condition of the fixed division of labour I. Comparing each decision variable of the mixed division of labour I and the fixed division of labour I in Table 1, the same conclusion can be derived. On condition of the mixed division of labour II, the production function of agent 1 specializing in producing good x satisifies  $f_{1x}(l_{1x}) = \alpha_{1x}l_{1x}$  and the production function of agent 2 specializing in producing good x satisifies  $f_{1x}(l_{1x}) = \alpha_{1x}l_{1x}$  and the production function of agent 2 specializing in producing in producing good y satisifies  $f_{2y}(l_{2y}) = \alpha_{2y}l_{2y}$ , which is the same as the condition

of the variable division of labour. Therefore, each decision variable of the mixed division of labour II and variable division of labour in Table 1 are the same.

The above studies proves that the production function setting like this possesses properties of both endogenous and exogenous comparative advantage, and this is why we call it mixed division-of-labour system. Each important individual decision variable and general equilibrium variable in mixed division-of-labour system are also reorganized in Table 1 and 2. According to Proposition 2.2, only if  $\frac{M_2}{M_1} = \frac{1-\beta_1}{\beta_2}$ , then individual good's value equates individual good's average labour expenditure.

In the following tables, VDL, FDL and MDL stand for the variable division of labour, the fixed division of labour and the mixed division of labour, respectively.

TABLE 1. Agent's Individual Decision

	agent 1			agent 2		
Division-of-Labour System	(Specializing in Producing $x$ )			(Specializing in Producing $y$ )		
	$x_1$	$x_1^s$	$y_1^d$	<i>y</i> 2	$y_2^s$	$x_2^d$
Variable Division of Labor	$\alpha_{1x}\beta_1$	$\alpha_{1x}(1-\beta_1)$	$\alpha_{2y}\beta_2 \frac{M_2}{M_1}$	$\alpha_{2y}(1-\beta_2)$	$\alpha_{2y}\beta_2$	$\alpha_{1x}(1-\beta_1)\frac{M_1}{M_2}$
Fixed Division of Labor I	$\beta_1$	$1 - \beta_1$	$\beta_2 \frac{M_2}{M_1}$	$1-\beta_2$	$\beta_2$	$(1-\beta_1)\frac{M_1}{M_2}^2$
Fixed Division of Labor II	$\beta_1$	$1 - \beta_1$	$\alpha_y \beta_2 \frac{\dot{M}_2}{M_1}$	$\alpha_y(1-\beta_2)$	$\alpha_y \beta_2$	$(1-\beta_1)\frac{M_1^2}{M_2}$
Mixed Division of Labor I	$\beta_1$	$1 - \beta_1$	$\beta_2 \frac{M_2}{M_1}$	$1-\beta_2$	$\beta_2$	$(1-\beta_1)\frac{M_1^2}{M_2}$
Mixed Division of Labor II	$\alpha_{1x}\beta_1$	$\alpha_{1x}(1-\beta_1)$	$\alpha_{2y}\beta_2 \frac{M_2}{M_1}$	$\alpha_{2y}(1-\beta_2)$	$\alpha_{2y}\beta_2$	$\alpha_{1x}(1-\beta_1)\frac{M_1}{M_2}$

#### 5. Conclusions and Future Problems

*Firstly*, This paper constructs a framework for the generalised theory of value by means of agents' two-stage decision approach, in which an agent at first makes his/her production decision to maximize total revenue, and consumption decision to maximize utility thereafter. The superiority in our newly amended approach over the original is that we endogenize configuration and structure of division of labour into individual choice behaviour but the original does not.

*Secondly*, in our general equilibrium setting, moreover, not only are marketclearing conditions taken into account, but also the allocation condition of labour force between production sectors of restrictive capacities, namely, *PERCA*.

*Finally*, in virtue of production functions' return-to-scale properties and endogenous/exogenous comparative advantage thereof, different division-of-labour systems are defined, and therefore described by this same theory of value which concludes that no matter which kind of division-of-labour system it is, the labour theory of value is only a special case of ours.

Nevertheless, models in this paper just investigate two kinds of good and two sectors of specialized production with labour input alone. How to expand this



 TABLE 2. General Equilibrium of Market Economy

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framework to n kinds of good and n sectors, as well as the determination of general value with non-specialized division of labour are two focal points of future study.

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